GEOMETRY IN MOTION. BETWEEN RESEARCH, EDUCATION AND OUTREACH

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Abstract
In this paper, we will explore the connections between mathematics and dance, understood as a movement of the body itself. The idea is to use the human body and the space that surrounds it in order to introduce important concepts of Geometry and, in general, Mathematics. More precisely, building on the experimentations carried out in 2015, we will discuss how body cognition can be a fundamental tool in communicating and teaching maths.

Keywords: Mathematics, dance, body, outreach, informal learning.

1 INTRODUCTION
In every type of dance there are many mathematical concepts used (sometimes) unconsciously both to find harmony of human bodies in the space, and for the logic.

In this paper we will explore the connections between mathematics and dance, understood as a movement of the body itself. The idea is to use the human body and the space that surrounds it in order to introduce important concepts of Geometry and, in general, Mathematics.

Our main two experimentation are a dance show called “Intrecci”, composed of four main choreographies, and a series of laboratories dedicated to different kind of students (from Preschool to University).

Intrecci was designed and implemented by ourselves together with the students of the dance school MiLaDance of Serra de’ Conti (Ancona, Italy). In the show, using the modern dance, we introduce and clarify the isometries of the space and the concept of homothety. Moreover, we work on the music, which not only acts as an accompaniment, but is an integral part of the show.

Then we present some different application on different kind of school (preschool, primary, etc.). In those activities we have kept in mind that the excellent learn path consists of the triad “do- see-symbolize” and that, as Emma Castelnuovo said to his french colleagues that criticized her method, it is not inappropriate “faire les mathématique avec le mains sales”.

The paper is structured as follows. In Section 2 we describe the theoretical framework of our study. More precisely, in subsection 2.1 we highlight the importance of the body in informal education and communication. Subsection 2.2 is devoted to the analysis of the benefits of exploiting astonishment while teaching/communicating, while in subsection 2.3 we discuss the utility of paying attention also to emotional aspects of learning. Section 3 provides an account of our experimentations: subsection 3.1 concerns the dance show Intrecci while subsection 3.2 provides a description of our laboratories of informal mathematics. the results of this study.

2 THEORETICAL FRAMEWORK
The theoretical basis of the experimentation described in the present chapter are essentially three: the importance of the body in informal education and communication; the benefit of exploiting astonishment while teaching/communicating; the utility of paying attention also to emotional aspects of learning.

2.1 Body as an instrument of education and communication
As researchers and experts on the topic, working with schools, science festivals, social events, etc., we often faced-off people who experience difficulties on the mathematical approach and reasoning.

As far as students are concerned, this fact is supported by several statistic surveys, among which we cite the Trends in International Mathematics and Science Study (TIMSS) and the (fairly discussed)
Programme for International Student Assessment (PISA). In both those tests the Italian students' preparation in Mathematics is insufficient and the Italian score appears at the bottom of the ranks.

Unfortunately this low competence does not affect just young students, and this feeling of aversion is common among all the Italian (and not only) population. Using Edward Frenkel words ([1]) “Intelligent people would never say, ‘I don’t care about art, or music’. But it is totally okay to say, ‘I hate math.’”. However, we think that at the base of this repulsion there must be real difficulties on the process of learning this subject, due to the effort for both the mathematical symbolism and for the gap between its abstract nature and real life. Indeed, all the thoughts and all the logical-mathematical structures are built in the child’s mind after these three phases: do -see -symbolize. Since the child learns and understands by doing ([2]), the body itself - together with all its expressions - becomes a useful tool for the learning (and communication) mechanisms. Nowadays the body is, unquestionably, one of the main characters of our culture: a remarkable amount of research is been carried on trying to build up a complex and well-structured vision of this physical dimension on different disciplinary fields. Nevertheless, we can not affirm the same for the pedagogy of the body. This definition can seem at least original and even difficult to understand. This can come from both historical and cultural reasons, such as:

- all the past educational theories have viewed at the body’s disciplines (and its movements) accentuating at the sanitary and healthily aspects and restricting the didactic view at the early years of childhood;
- the body is the unique object that, in order to be observed and studied, can not be separated by ourselves and put at a good distance. The body is our guarantee of existence, entity that belongs to us and to which we belong, obligated cohabiting and everyday travel companion. [...] space that imposes its own measures ([3]). Studying it, we should take into account our inevitable subjectivity and emotions, which represent the corporeality in a more complex and paradoxical nature ([4]).
- both Plato and Descartes underlined a marked separation between the soul (mind) and the body. Since then, according to Science, the body, separated from its spiritual part, is relegated to be just an object.

Those ideas are well entrenched in the common way of thinking. On the other hand, since the early years of 1900, we can find research on the centrality of the body and its movements on the educational processes. Maurice Merleau-Ponty, a 20th century French philosopher, came to the conclusion that the body can not be just an object, but it becomes the necessary condition for the human beings to the experience ([5]). The body is the perceptive window through the entire world and the main vehicle for the concept of being alive. Furthermore, educational processes that do not take into account the feelings during learning seem to be ineffective, and experiential didactics can be capable to transmit knowledge during the becoming life of the students. Corporeal training during education processes takes care of the real existence and works for the competence of being. Maybe those thoughts could help for a new planning of the school systems, but often they are obstructed, starting from the University level, where the professional teachers have to be trained. Reading the current Ministerial programs for the Primary School, we notice the awareness of the peculiar role of the body (and its movements) in the cognitive processes. From 1985 the name Physical Activity has been changed into Physical Education (PE), emphasizing its educational potential. The explicit reason, that can be read in the programmes, is that “…the method learned by means of the motion skills is internalized to form what we call thought “. Clearly, we can not require that this huge task is done simply by a single school subject (which has just two hours per week, per class). Furthermore, for someone “physical” education is just a synonym of “gym”: even if its name recalls the ideal unity of body and mind, in reality those people interpret it as merely regarding the body, seen as a set of functional parts which need training.

As we mentioned before, the difficulties in Mathematics do not stop after school time, but they turn into barriers against the topic. Thus this method can be convenient also for outreach activities addressed to a general public. Moreover, we know that non-verbal communication plays a crucial role. Studies reported in ([6]) state that only the 7% of the perception that the other people have on our arguments depends on the content that we actually say, while the 38% has to do with the tone of our voice and the 55% depends on the so-called non-verbal communication. This latter has to do with the distance that we have with other people, the posture we take, the way we move our hands and our eyes. If we want to establish an emotional contact with our interlocutor, it is important to adopt certain - verbal and non-verbal - behaviour, which should facilitated the empathy. As Paul Watzalvick says in [7], the body
plays a central role also in the communication processes (similar to the educational processes that we treated before). Indeed: “We can not non-communicate; the behaviour does not have its opposite, thus a non-behaviour. [...] The activity or inactivity, words or silence, they all have an inner value of message: they influence others, which in turn can not non-answer to these communications and in this way they still communicate”

We can conclude ([8]) that the body is the intermediary of the knowledge and the protagonist of communication, as a medium for the organization of the relations and a privileged tool for the promotion of the psychophysical wealth of every person in his uniqueness and specificity.

2.2 Feel astonishment for the Mathematics of every-day life

Mathematics teaches us, starting from its language, the concept of eternity, truth and coherence ([1], [9]). The mathematician, like an artist, embodies the character of a creator who is free to move, to invent, to choose, to express himself. Unfortunately, people are often completely unaware of this. Moreover, the lack of preparation of our country in Mathematics, perhaps due to the Gentile’s impression that it is a “barren and empty” subject, comparable to a “stone” ([9]), is assessed by many tests all over the years. Such a chronic problem has to be faced with all the possible means, first of all the training of the population, starting from an early age.

Many people understand the real need for Mathematics only once outside School, bumping into real problems at work, for which its knowledge is necessary. Such a re-discovery of Mathematics seems then to be due mainly to the various applications it has in all the other sciences. In our opinion, this is a rather short-sighted view, because Maths is much more than that: besides being the language of other sciences, it has a creative, inventive and daring power that people should know, and mathematics should try to communicate. The first task required to the educators and experts on the topic is to demolish all the educational and not-educational stereotypes. In [10] - Artigue was, in that time, president of the International Commission on Mathematical Instruction - we read: “An high-quality education in Mathematics should enable to forge a positive and appropriate image of Mathematics. Thus, it must be accurate both for Mathematical concepts and practices. This education must let the students to analyse the needs to which Mathematics replies and also that this subject has a long history, combined to the history of humanity.”

Furthermore, the mathematical literacy enables each of us to understand a situation, to reason, to analyse and criticize data. This gives rise to critical and logical thinking of any choice, basing this action of mind on the full comprehension, on the development of models, on the prediction of the results, on the control of the actual choices. This knowledge is more and more necessary (and required at work) to build a society of increasingly autonomous individuals. We wonder about the reasons why there are so many stereotypes in Mathematics, especially in the School world, and of a different nature: in the manner of speaking of Maths, in the ways of doing Maths and in the ways of thinking about Maths. In primis, we thought to find the answers at School: what is the idea of Mathematics that Maths teachers have? Too often they limit themselves to re-dust their school knowledge, proposing old memories (most of the times reinterpreted and maimed) and mechanical activities, being afraid of innovations that they consider to be difficult to manage. Unfortunately, this mechanism starts since Preschools. Moreover, unlike other subjects, learning places for Mathematics rarely differ from School. Indeed, in some cases, it seems clear that the Maths that you learn in School is totally different from the ones that you can meet in the street, at home, at the museum, etc.

In order to improve this situation, some of the most important Italian centres of education in Mathematics (Gruppo di ricerca e sperimentazione in didattica e divulgazione della Matematica, Nucleo di ricerca e sperimentazione in didattica della Matematica in Bologna and matematita ) call into question the methods of teaching, in order to favour those who prefer a spontaneous learning done by the child. An a-teaching situation is definitely to be favoured compared to a teaching one: with such an expression we mean moments when the proposal made by the educator does not have an educational explicit expressed goal. In this case the student (but also the teacher) decides to get involved in an activity without knowing the cognitive purpose that has to be achieved. These kind of activities appear to be fully implemented in the Preschools, while they seem to be extremely difficult to carry out for older students (probably due to the so called didactic contract ([11]).

In the preschool the child it is not (yet) subject to the evaluation, therefore he is free and able to approach the knowledge in a more personal way, without receiving back from the educator a negative judgement. This approach is the same that we have in a generic public that participates in some outreach activities. Basically, we have to specify that there is a substantial and clear difference
between the purpose of the communication and the purpose of the teaching. If the first aims to intrigue, de-mystify, entertain, in order to create a fertile humus for a specific future learning (which can and must take place in different contexts and with different times), education aims to teach. This difference is emphasized when the order and degree of the School raises. On the contrary, it is quite nuanced in preschool, as we can see by the national objectives as stated in [12].

Using Di Paola [13] words “To activate the connection between the developing mind and content organization in preschool’s educational setting is similar to enter into a very special path, that requires special considerations compared to those generated by the activation of the same report in later grades of education. It does not require transmission of knowledge, nor specific learning related to specific elements of knowledge codified by several disciplines. The reference to the content is [...] in a proto-disciplinary key and then [...] the content can be called Proto-mathematics”.

With this in mind, it was quite natural for us to start our project from the preschools, where we expected to communicate - and not properly teach - this kind of proto-mathematics. According to the statement above, working with 3 to 5 years-old children let us move into a proto-disciplinary context: such a context is very familiar to us, as it is typical of scientific communication. Also, as mentioned above, while working with very young children we aim to create a fertile humus for the subsequent formalization of induced proto-concepts, against the development of preconceptions: this is indeed the purpose of Scientific outreach for the generic public of any age. Interacting with young children, we also noticed that their own way of learning a proto-concept relies on the same main feature of the understanding of the generic public in a well-done outreach event: the astonishment. Astonishment is the impulse of knowledge, the condition of thought [14], the door of artistic, technical and scientific understanding of reality. Etymologically (from the Latin stupor, from the verb stupere that is surprise) the astonishment is associated with the reaction to something unforeseen and unexpected, from which we are fought and beaten. “Something that breaks from the outside. This is the supreme method of knowledge. We must restore to the event its ontological dimension of a new beginning. It is an irruption of the new, that breaks the gears, and sets in motion a process” (15). It is a well-known anthropological condition, and as such it cannot remain far from one of the primary human experience, that is the experience of learning. It seems appropriate to create a sort of triangle, which links the astonishment to the concepts of surprise and discovery. Can the School be an environment in which experiencing surprise and discovery? If so, can this be an advantage to the prospect of learning? Undoubtedly, school education has the task to foster the acquisition of knowledge. However, education cannot be limited to this. One of the last diffuse reflections speaks about the current society as the knowledge society [16]. The current citizen is established by the experience of knowledge, and a great emphasis is given to the ability to learning throughout life (long life learning). Of course, here we put the accent on the general knowledge, rather that on a specific, disciplinary one. Here it appears the curiositas, which is a fourth point that we can add to our triangle. Furthermore we are now facing an epistemological mutation that made liquid [17] all the boundaries between different knowledges and less analytical and straightforward the experience of verbal and non-verbal knowing, which is always open to new developments. School cannot ignore all this and can not simply transmit knowledge without thinking about its characteristic to be a permanent attitude. Indeed, today knowledge has these two peculiarities of surprise and discovery: in a metaphor, this is the crossing of Hercules’ Pillars, and School is called to create new Ulysses. Now, the huge question is which are the most appropriate educational conditions for which this can happen? Educate and train to astonishment becomes necessary inside and outside School. The astonishment is like having eyes wide open on the real world, and it’s not a surprise that it was defined by Heidegger as the desire to see. Educating to astonishment has several different aspects: training to attention, developing the curiosity, promoting intrinsic motivation, and promoting the contact with the reality.

Speaking, more precisely, on the methodologies, we would like to mention Dewey’s, one of the best that fits those instances. According to him, the connection between learning and experience is not immediate. Thus the learning by doing may not be turned in a mere un-reflective procedure, but it needs a real reflection on the act of experience, on its significant features and the possibilities of modelling. The greatest hope is then that this cognitive dimension is not relegated only to an early age, and that such an emotional momentum to the discovery never ends in a human being.

2.3 Emotional intelligence

As a final theoretical consideration regarding our research project in Didactics of Mathematics, we want to deal with an innovative, subjective and emotional change that became established among the different educational methodologies. Borrowing on loan the words of the German theologian Dorothee
Solle, the answer to the question “How would you explain to a child what is happiness? “ is “I would not explain it to him, I would give him a ball, so that he could play with it” ([18]). In this gesture there is a collection of reasons and educational purposes directed to a true promotion of the individual-child, encouraging him - from an early age - to play an active role in the never-ending path of knowledge. Moreover, the words emphasize the inestimable value that the child can draw from the active experience, rather than from passive actions: in this specific instance from an extreme emotional level (give to a child a ball) to the cognitive level (understand what is happiness). In our opinion, this method is not linked to a specific age, even if adults have barriers risen by social contacts, that instead are not present in children. Stating ([19]), “the wellness of the cognitive system allows the emotions to have a more appropriate dimension, while the wellness of the emotional system allows the cognitive system to fly “. Thus, learning in a simple static and passive process is insufficient because it is an action disconnected from reality and direct experience. The emotional aspect makes learning not only more enjoyable, but also more effective ([20]). At preschool level this feature becomes evident because every little piece of the child’s cognitive puzzle rises in a recreational and active way: most of the time children are involved in first person and they learn while they are having fun. As it was widely said, during the transition to the Primary School, the emotional approach is, mostly, lost, favouring the impersonal approach. The laboratorial didactics tries to stem such an aseptic drift. A mention is due to the Gardner studies on the Theory of Multiple Intelligences ([21]), together with the Theory of Emotional Intelligence by Goleman ([20]): they both tend to demolish the monolithic conception of a single intelligence, in favour of the existence of multiple intelligences, each quite autonomous and present (in different amount) in each individual. Clearly, if we take into account those theories, some of the fundamental cornerstones of the traditional school careers would not have any more a scientific evidence. When, for instance, a student is considered to be good? We can agree that a good student is one who is successful in most disciplines. Nevertheless, in order to give a sense to the expression “successful in most disciplines”, the classical approach tends to make a sort of quantitative media, placing at the last rank of the scale disciplines as music, dance and, in general, all those related to physical activities. On the contrary, Garner’s approach gives equal weight to all the abilities, that are classified according to those “intelligences”: musical-rhythmic and harmonic intelligence, visual-spatial intelligence, verbal-linguistic intelligence, logical-mathematical intelligence, bodily-kinesthetic intelligence, interpersonal intelligence, intrapersonal intelligence. According to this theory, the concept of intelligence is to be understood as a special skill of the individual. Although those are more or less innate in each human being, they are not static and they may be trained and eventually they can decay as time passes. Gardner himself mentioned that listing all the manifestations of human intelligence is a too complex task, since each macro-kind contains various sub-kinds of intelligence. According to his theory, a more self-tailored vision of learning is needed. Moreover, Garner’s approach emphasizes the need of self-consciousness in order to favour those learning systems that better fit the peculiar intelligences of each individual. Of course, this approach has as consequence the fact that learning (and teaching) has to be highly personalized, not only with respect to the abilities but also with respect to the time it requires. Quoting Kundera in Slowness ([22]): “The degree of slowness is directly proportional to the intensity of memory; the degree of speed is directly proportional to the intensity of forgetting.”.

The teaching methodology that better fits those ideas is, without doubt, the laboratorial, where the child tries and verifies alone - or with a group of his peers. At the base of these experiences there is John Dewey’s active pedagogy: School should exercise intelligence on concrete experiences of reality, contextualizing it and globalizing it, and it should carry out social experiences of a democratic (not authoritarian nor selective) kind. Several educational models derive from this pedagogy: Maria Montessori, Ovide Decroly, the Rinnovata Pizzigoni, the Reggio Children and Scouting of Baden Powell. They all agree with the idea that laboratories are connection between activities of study and reflections on life-experiences: laboratories take into account science, art, technology and the border line between mind and body, provide the child with a strong critical sense and make him use every available sense ([23]).

The educator is asked to devise a psycho-social situation that represents an incentive for the student to know himself and integrate knowledge in an original construction, consistent with the Montessori’s vision “help me to do it by myself ” and on the same wavelength with the new interdisciplinary field of Embodied Cognitive Science, whose purpose is to explain the mechanisms underlying intelligent behaviour according Goleman ([20]). The laboratorial proposal, which is of course adaptable to any subject, becomes even more fitting if the subject to be taught is Mathematics. As taught by Emma Castelnuovo ([24],[25], “Mathematics is understood even with hands”. Obviously teachers’ choices make the difference. Specifically Maths teachers have to space themselves out the goal of doing
Mathematics, through the broader goal of exposing students to the widest possible panorama of mathematical facts ([26]). The teacher is responsible for planning and designing learning paths.

As we mentioned above, and also from the more-than-10-years experience of the matematita centre, we know that the laboratory approach is an interesting possibility that the teacher must take into account, both for class and for its continuous preparation and education. In the specific preparation of our experimentations in the schools we take the theoretical inspiration from the matematita’s definition of “laboratory”, that is an activity where:

- students have an active role. Thus, they should work concretely to build their knowledge;
- teachers have the role of expert guides that look, listen and respond to any questions, and help the students to summarize the activities. Possibly technical experts accompany them.

According to this last point it is fundamental to do an initial training for teachers, aimed to highlight both the direction of the class during the laboratory and the choice of the mathematical concepts for the activity. Those concepts must not have an immediate solution and they should not even include non-calibrated techniques. They should mainly require common sense and proper thinking - the basic ingredients for any long-lasting acquisition of a mathematical concept.

Finally, there are four basic keywords that can be useful in a practical implementation of a laboratory ([27]):

- accuracy: it means to clearly express each step of the activity, explicating all the theoretical concepts needed. This of course takes time, but it this time is well spent in a generic vision of teaching and learning. During the laboratory, the child is placed in front of a problem with real and tangible difficulties that realize the theoretical point. This accuracy is never artificial, but it becomes essential and helpful. Concerning language, it can be useful, especially with young children, to use at first a simple and familiar language, and turn into a more specific terminology only when the mathematical concept is fully understood;
- error: the role of the error is absolutely precious and, indeed, it is the laboratory’s driving force. Teacher’s task is therefore not to instil in children (for the teaching contract) fear of failure - one of the most powerful enemies of learning. Through the error, new mechanisms both for teacher and students can arise. Definitely, linked to this point is the concept of evaluation of the laboratorial activity (see below);
- discussion: during the laboratory, children have to discuss with each other, making the effort to make their personal vision clear to the others. Obviously the child succeeds in this task only after gaining himself a full awareness of the topic. It is also necessary to point out that similar peer-discussions can happen only during these kinds of activities, as in a normal lesson there is always an unbalanced communication between the teacher (who knows) and the class (who does not know). The strength of a joint informal communication leads the children to understand if they have understood. In this final stage it is necessary and useful to have the teacher’s guide that summarizes the salient features of the path of knowledge done by the class;
- evaluation: the workshop activity, by its nature, should not be directly subjected to an evaluation. In case, its outcomes can be evaluated (in a medium-long term) or maybe it is possible to evaluate globally the whole laboratorial activity done by the class.

3 TURNING THEORY INTO PRACTICE

The present section lists two of our experimentations: the dance show Intrecci and our laboratories of Informal Maths.

3.1 Intrecci

In every type of dance, from the traditional and the classical ones up to the modern, there are many mathematical concepts. Some of them are unconsciously used, both to find harmony of human bodies in the space, and to determine harmonic interactions. In contemporary dance techniques some choreographer noticed the strong presence of Maths and decided to take inspiration from it. Intrecci is a dance choreography, showed in preview at the Conference “Incontri con la Matematica, n.29” (November 2015), in Castel San Pietro Terme ([28]), and then filmed in a video (150 minutes long)(see [29]). In the show, topological and geometrical elements are used not only for helping the public to “see” some abstract object, but also to make the audience aware of how Maths is present in
Words, scenography and images help the audience to focus the attention on the dancers’ lines and bodies. The music has been chosen with attention, in order to exalt choreographic elements. With this project, described in detail in [40] and [57], we explore the connections between Mathematics and dance, understood as a movement of the body itself. Starting from the perception of the body in the space, every dancer has to be perfectly awake to understand his role in the tri-dimensional space. Indeed, every choreography is composed by lines, volumes, geometrical and symmetrical figures which move in the space according to a specific sequence. We decided to use contemporary dance techniques for two main reasons: first of all, this type of dance starts from the study of everyday life movements; in fact, according to the German choreographer Pina Baush, “To understand what I am saying, you have to believe that dance is something other than technique. We forget where the movements come from. They are born from life. When you create a new work, the point of departure must be contemporary life - not existing forms of dance.” Another reason is that this type of technique gives freedom of movement and it allows creating different lines and volumes with dancers bodies. Wayne McGregor, Karl Schaffer and Erik Stern also inspired our work. The choreography Intrecci is performed by a group of young dancers (12-19 years old), students at the MiLaDanse Dance School in Serra de’ Conti: a member of our research group (Ilaria Giancamilli) works there as a dance teacher, and we “used” her pupils to observe the reaction of the dancers themselves towards the relationship between Maths and Dance. The show is divided into four main parts - each of them with a specific music and theme.

The fist is called Identity. Here we tried to explain who is a mathematician. We started asking ourselves who we are, who we want to be, what kind of researcher we want to be. Then we posed these same questions to the dancers: what kind of ideas could they have on the character of a mathematician? Our and their answers were different, but we find somehow some point of agreement. A mathematician is a curious and methodic person; he/ she questions himself/ herself and wants to go over his/ her limits. The next step was to associate to every characteristic a movement, collect all the movements and write a story with them. The music chosen was the animated poetry Delirio matematico1, created by Bruno, Ciampa, Ciponte (Delirio matematico: https://www.youtube.com/watch?v=uVn71OyGzVk).

We chose it because we thought it could predispose the audience to see Maths not as a scarred subject, but as a science close to our life and our humanity.

The second part is called Phi. Here we worked with symmetries: we based the sequence of movements on numerical series and we showed the construction of the aurea section starting from the aureo triangle. For the music, we choose a track from Einstein on the beach2, by Philip Glass and Robert Wilson: its title is Knee n°5, and we selected it essentially for the atmosphere it creates.

The third part is called Move. With this one we wanted to illustrate geometrical transformations such as translation, rotation and homothety. Indeed all this composition is based on the movement of a square in the plane and then in the space. In this case we choose a song, Body percussion di the percussion show, in a peculiar kind of music, called clapping music, for different reasons: first of all, clapping music structure is based on a repetition of a series of sounds made by body, and this gives the feeling of something that develops in a linear manner (more easily interpretable thanks to simple modular arithmetic notions); moreover, it is very joyful and complies with the age of the dancers who interpret the choreography.

The last part is called Twine. We started with the representation of inverse functions: if we imagine every movement as a function which warps the dancers’ bodies and the figure of the corps de ballet, we can perform the inverse movement to return to the original position. We inserted also knots and brides in the choreography, not only for creating scenic figures, but also for helping the audience to visualize these abstract mathematical objects. We wanted to emphasize the dancers’ movements, so we choose a discrete but emphatic music, Metamorphosis 1 by Philip Glass.

Martina Carrera makes costumes. For tutu she created a Möbius strip.

Those models were explicitly thought to fit the choreographic demands, and they are in fact a fundamental ingredient of the choreographies. The peculiar genesis of this choreography led us to rethink it as a starting point for laboratories or outreach events on Maths and body.
3.2 Laboratories on informal Mathematics

As we mention in the previous section, the idea of the choreography led us to conceive a series of didactics experiences strictly connected to it, in terms of modalities, topics and aims. Here we describe the experience made in Preschool - Scuola d’Infanzia in Serravalle del Chienti (MC) (ISC Camerino “U. Betti”). We were interested to transpose in a scholastic context the investigation on the body as a vehicle of communication of basic concept of Geometry, first explored in a dance and show situation. For our experimentation we organized three weekly meetings from the 28th of April to the 12th of May 2015. In this Preschool there is an unique class, made up of 18 children: 4 of 5 years old, 5 of 4 years old and 9 of 2.5-3 years old. Many things have been said about the unique-class, surely in a much appropriate context. We don’t want here to analyse the pro and con of it, neither the opportunities that it gives to the children engaged. For us it is sufficient to say that, in our specific project, we had the impression that working in a unique-class in a preschool was much more exciting and productive that it would had been otherwise. Indeed, the group of the older children, voluntarily, acted as a bridge between the experts (teachers and us) and the younger children: they translated the activities in a more direct language (education by peers). Moreover we have to mention that this class was already motivated to experience those kind of scientific activities: one of their teacher is graduated in Mathematics and she organizes weekly activities related to science. Thus we can say that this class is somehow special and particularly receptive to this type of activities.

The first meeting is called “Harmony and symmetry” and it has the aim to identify harmonic proportions and symmetries. The proto-mathematical concepts transmitted are: space, order, dimension, shapes, spatio-temporal changes. After having met the class and introduced ourselves, we gave the children lighter, stronger and unbreakable plexiglass mirrors. We left time for children to play with these objects and to understand how these objects work. After 150 almost all the children were capable of explaining the rule and the law behind the object-mirror. Then we started to use the mirror in order to understand the symmetries in the objects around us and those symmetries that are in our body. Finally we played the dance of the mirror: the class was splitted into two parts. Half of them could move freely and the others had to be mirrors: every child-mirror should imitate the behaviour of the child-object in front of him.

The second meeting is called “Points and lines” and it has the aim to communicate the concepts of point, line (broken, continuous, curve...), rotation and translation. The proto-mathematical concepts transmitted are: movements in the space, closed and open curve, and connection of a space and practicability of a path. We put in the floor of the room little red disks (that later on were called nest). We asked each kid to occupy one nest, as a little bird. Then we started to propose a series of different rules that have to be used in order to go from a point (disk-nest) to the other, associated with different songs. Then, we gave all the children a beak and we introduced the concept of translation and rotation. Finally we moved to another part of the room where there was a big white poster. We asked the children, wearing plastic-socks soaked in red tempera, to walk in different ways. At the end we watched the tracks of the different walking paths left on the poster.

The third meeting is called “Knot” and it is aimed to make the children discover the concept of geometric knot. The proto-mathematical concepts transmitted are: movements in the space, closed and open curve, and connection of a space, shapes and misures. In this last meeting we focused on how to use the body in order to discover the theory of knots. First, we asked the children to draw a knot and describe with word how is it. Then we started to play creating knots with the children bodies (human knots): we asked them to get very close one to each other, then each child had to tie his hands to other hands of two different children - possibly far from him/her. The result is a human knot, equivalent to a Hopf Link, impossible to simplify in R^3. The only possible way to get untied is to simulate a cut: one child move the rope into the wrist of the other child and thus he/she unties the knot. A great attention was given to the choice of the materials that to be used during the laboratories, that had to be cheap and recyclable. Whenever it was possible, we used objects that children had already viewed and used in their classroom (or that they could build by themselves).

Each meeting was characterized by a well-specific object: a mirror for the first, a red disk for the second, a rope for the third. Those materials, however, were only used in order to help clarifying the proto-matematical and proto-geometrical concepts: the true protagonist of the workshops was the body itself (the space that it occupied and the movements that it made).
REFERENCES


[34] Intrecci’s video https://vimeo.com/168188218 (password: intrecci).