CHILDREN'S ABILITIES IN THE AREAS OF COMPARISON, ARRANGEMENT AND WORK WITH A WHOLE

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Abstract
Comparison of objects, their arrangement and work with a whole and its parts are among activities that children at 3-6 years of age should be able to deal with. A pilot study took place in several nursery schools in the Czech Republic, whose aim was to assess the skill level of children of four age categories (three-, four-, five- and six-year-old children) in the areas of object comparison, object arrangement and work with a whole. Three different activities of increasing difficulty were prepared for each of these areas. Children thus solved nine tasks in the course of three weeks. Tasks focused on inherent comparison (finding differences in pictures), basic comparison (comparison of weight), chronological arrangement (arrangement of cards with pictures according to time progression), qualitative arrangement (arrangement of objects according to weight), composition (assembling a picture from its parts), completing (finishing a half-drawn picture) and reproduction (construction of toy bricks according to a printed plan). Teaching aids or work sheets were created for these tasks. Experiment was carried out by observing children completing single activities. A video record and photodocumentation of the activities were made. Based on quantitative and qualitative analysis of the pilot study it was discovered that for the examined sample of children, the most difficult activities were arrangement and orientation in time. Comparison appeared to be the least difficult activity.

1 INTRODUCTION
Before a preschool child (3-6 years old) starts forming her/his so called pre-numeric and geometrical conceptions, it is necessary for her/him to develop many skills and abilities which are understood as prerequisites to master school mathematics. Basic mathematical conceptions are developed in the course of every activity taking place at nurseries or in domestic environment.

The following abilities are among pre-mathematical skills a child should have acquired before starting school attendance: conclusion drawing, comparison, assignment, arrangement, assortment, orientation in space and plane, orientation in time, the whole and its parts, elements of probability, elements of combinatorics, elements of topology, propositional logic and negation of a statement. All types of pre-mathematical skills penetrate all everyday activities. Every activity cultivates only a subset of these skills but typically includes more than one.

We are going to focus in detail only on three selected areas: comparison, arrangement and work with a whole and its parts.

2 COMPARISON, ARRANGEMENT AND WORK WITH A WHOLE

2.1 Comparison
Comparison is a process of finding a relation between two objects or phenomena. During this process it is necessary for a child to have a developed ability of perception and the ability to recall two objects at once. In the event of the two objects being too big or too remote and when the observer examines one object at a time, an auxiliary procedure for easier comparison must be found. Preschool children can compare three-dimensional physical objects (articles, persons, animals), two-dimensional objects (pictures, photographs, shadows) or sounds (stamping, clapping, talk, music) (Kaslova, 2010).

At nursery, children work with several types of comparisons. Natural comparison is based on finding concordances and differences. The child seeks an answer to the question: Are these objects equal in all aspects? The most common activities are finding differences between two pictures or finding two identical pictures in a set of many.

Basic comparison takes place following specific rules. We can compare amount, numbers, length or weight. When comparing amounts, we put compared objects to couples, we do not state the number
of objects in single groups but we choose from a set of relationships (same as, greater than, smaller than). Thus, if a child works with pictures of puppies and dog beds, he/she only assesses whether there are more puppies or beds. If the child compares numbers, he/she has to determine the number of puppies and dog beds and compare these numbers (e.g. “There are five puppies and three beds. Five is greater than three.”). When comparing length the child works with two crayons of different colours and finds out which crayon is longer/shorter. Weight comparison is usually carried out by the child picking up both objects at once.

Reduced comparison is very close to basic comparison. It is about choosing one of the relationships and asking yes/no questions to the child. Thus the child is given two briefcases of different colours (yellow and green) and different weights and is asked: “Is the yellow briefcase lighter than the green one?” The child answers with just yes/no or with a whole sentence.

Comparison by difference is based on finding relations between two objects. We work with a “by how much” relation, in the sense of “Compare by how much two objects differ”. Depending on the child’s answer, we distinguish two stages. First stage is called proportionality. The child points on compared objects and replies “by this”, or instead of gestures he/she can use words (by a bit, by several, by nothing). At the second stage, the relationship is expressed using a number; the child describes the difference in a concrete way: “By one marble, by three dice, by two beads.” (Pechouckova, 2015; Kaslova, 2010).

2.2 Arrangement

Arrangement is both a procedure and its result at the same time. By doing an arrangement, the child cultivates orientation in diverse impulses and facilitates memorising. Arrangement can be classified according to two criteria: the mathematical viewpoint and the contextual viewpoint (i.e. children’s viewpoint).

From the mathematical viewpoint, we distinguish a sharp linear arrangement, unsharp linear arrangement and natural arrangement. In the context of sharp linear arrangement, every object from a set has its unequivocal placing or ranking. Thus, there are no two or more objects in the set that would have the same monitored characteristics and would be at the same place (the child is supposed to arrange six cups according to their size starting from the smallest one; no two cups have the same size). In the unsharp linear arrangement, there are two and more objects which are impossible to determine order for. In the case of arranging cups there would be two cups of the same size in the set. This type of arrangement is difficult for children and is not used in nursery education, as well as natural arrangement, which is related to relationships between numbers and is suitable only for primary school mathematics.

Arrangement classification according to the context is based on the children’s viewpoint. We distinguish time arrangement, space arrangement, space-time arrangement and quantitative arrangement. Time arrangement is determined by relations between events. Children can arrange cards with pictures from a known tale according to the plot unwinding. Space arrangement is about the mutual position of objects in plane, space or line in relation to a target. When flying paper kites of diverse colours, children arrange the kites according to the height they have reached (from the kite that is flying the highest to the lowest flying kite). Space-time arrangement is understood in time and space at once. The children can arrange themselves in a line according to when they came to nursery that morning. Quantitative arrangement is related to number or amount of objects. Children can arrange chimneys according to number of wooden blocks they are made of – from the chimney with the smallest number of blocks to the one with the highest number (Kaslova, 2010).

2.3 Work with a Whole

In the first stage of their development, children see an object only as a whole and do not perceive anything else. Later on, children start realising that a whole consists of single parts. This phase comes when the child starts decomposing the whole into its parts. At the beginning, decomposition of a whole is interconnected with experimenting. First, the child decomposes only small articles, later he/she starts with larger ones. There is no pre-set end or time limitation for decomposition, there are no given rules. What matters is whether the child has fun and whether he/she tries to decompose further and further. Decomposition of a whole has two specific aspects – destruction and division into two parts. Destruction means destroying the original whole which cannot be re-assembled. The whole is destroyed, torn to pieces, broken, it does not work. Destruction sometimes serves as a relaxation technique, related to venting emotions. During division into two parts, the child sets one
part aside and continues with the other one, either decomposing it further or playing with it. Typically the child does not try anymore to connect both parts together and re-create the whole. Division into two parts has its use in school mathematics, e.g. in decomposition of numbers.

After the decomposition stage the child passes to the **composition of a whole**. If there are no given rules for the composition, it is a free composition (e.g. child can paint or build from blocks whatever he/she wants). It is related to enjoyment and aesthetic perception. If there are given rules for composition which are related to a reality (what we have seen), a context, a model (plan attached to a toy construction kit) or a vision, it is a bound composition. When doing composition, we distinguish four specific versions. **Completion** means that a certain part of a whole is given and children only complete it to create a whole. **Re-composition** is a repeated composition of a whole. We start with a whole, decompose it to its parts and re-assemble it again. The whole can be changed, then decomposed and recomposed again. The difficulty level increases with every change of the whole. The entire process does not need to take place at once, the child can decompose the whole one day and re-compose it a day after. During **reconstruction** the original whole is decomposed and then changes are introduced during the re-assembly based on memory. **Reproduction** represents composition of a whole following precisely a model without any deviation. The exact look shown on the model must be preserved.

3 EXPERIMENT WITH CHILDREN

A pilot study, whose goal was to compare the level of skills in the areas of comparison, arrangement and work with a whole of children in four age categories (three-year-olds, four-year-olds, five-year-olds, six-year-olds) included a total of 80 children (20 children in each category). The experimenters worked with every child individually. A fairy tale, which was read to every child, served as motivation for all given tasks.

3.1 Task No. 1: Comparison of Two Pictures (Natural Comparison)

The child has two pictures in front of him/her. His/her task is to find four differences between them (Fig. 1, Fig. 2).

![Figure 1. Comparison – differences.](image1)

![Figure 2. Comparison – differences.](image2)

This was very easy. All children solved it quickly. The longest time needed to solve this task was 3 minutes. Three-year-olds struggled to find a tiny difference that was not visible at first glance, a missing “ball” on the right side of the throne. Four-, five- and six-year-old children had no difficulties to solve the task.

Children of all categories chose the same strategy for finding differences. Their look was first attracted by colour differences of old woman’s headscarf and king’s slippers. Then they found the orb and finally the above described ball.

3.2 Task No. 2: Finishing Drawing of a Castle (Completion)

The child has a grid in front of him/her with half of a castle drawn in it. His/her task is to complete the drawing so that both sides of the castle are axially symmetric.
Completing the drawing was time-consuming for all children. It was obvious that all children needed enough time to solve the task.

Before getting to the actual task, three-year-olds were given a master drawing with the complete castle. Children did not connect the master drawing with their worksheet and verbal aid was needed using questions: “What does this picture (master drawing) remind you of? What do you see in your sheet (worksheet)?” The master image was a “castle” to children, image on worksheet was a “house, castle”, but a completely different one. Therefore, they could not link the master drawing to the worksheet and they did not realise that there is half of the master image drawn on their worksheet. After ten minutes, they pushed the papers away and did not want to continue the task.

Neither four-year-olds completed the task. They were also given the master drawing before starting. Children in this age category drew various lines or even a sun to the worksheet, they added a frame to the sheet, but they did not even start completing the castle. Also these children were given verbal aid with questions (see above) but they could not understand the link between master drawing and worksheet.

Five-year-olds completed the task with some help, first using the master drawing to see the whole castle and then verbal aid using the above-mentioned questions. Some verbal aid was needed when colouring in the drawn half. Children’s attention had to be drawn to colours used in the printed half and they compared them with colours of their crayons. Six-year-olds managed the task without difficulties. Immediately after being given the worksheet, they started completing the picture exactly according to the printed half of castle. They also chose colours on their own.

3.3 Task No. 3A: Weight Comparison of Purses (Basic Comparison)

There are three purses (bags) of the same size on the table in front of the child. His/her task is to compare the weights of single purses and find out, which one is the heaviest, the medium heavy and the lightest.

The weight comparison was more difficult for children than natural comparison in the first task. Nevertheless all children solved this task within ten minutes. It was necessary to emphasise to three-year-olds that there were three purses, that they had to compare all of them and find the heaviest and the lightest ones. They would catch the same pair of purses repeatedly. They would keep the same purse in one hand and exchange the remaining two in the other hand. They struggled to find the heaviest (lightest) purse. They solved the task only after verbal aid. Four-year-olds approached the task similarly to three-year-olds. It was again necessary to remind them how many purses they had and what their task was. Once they understood the task they solved it without further help. Children at the age of five and six were autonomous at solving this task. Two strategies emerged:

1. From the first compared pair of purses, children put the heavier one on the right-hand side of the table. From the second compared pair, they put again the heavier one on the right side. They put the third purse, which they concluded to be the lightest one, on the left side. In the end, children compared the two “heavier” purses. Five children chose this strategy.

2. This strategy is similar to the previous one. Three children put lighter purses on the right side of the table.

3.4 Task No. 3B: Arrangement of Purses by Weight (Sharp Linear Arrangement)

There are three purses (bags) of the same size on the table in front of the child. His/her task is to arrange them in a line on the table from lightest to heaviest.

The biggest difficulty of this task was the arrangement of all three purses. Three-year-old children did not understand the phrase “from heaviest to lightest”. Although they managed to determine which purse was the heaviest or lightest one, they did not understand the aim of the task and they did not know what to do. After some time they stopped focusing and cooperating. Children at the age of four and five years arranged the purses correctly but they described them as “heavy – heavy – light”. It was necessary to explain them the terms heavy – heavier – the heaviest, light – lighter – the lightest and show it on real examples. Six-year-olds ordered and named purses correctly without assistance.
3.5 Task No. 4: Assembly of a Picture (Composition)

There are pieces of a picture from a known fairy tale spread on the table (jigsaw). The child’s task is to assemble the picture from its pieces (Fig. 3).

![Figure 3. Assembly of a picture.](image)

When assembling the picture it was obvious that children already had experience with assembly (jigsaw) from a small number of pieces. Three-year-olds and half of four-year-olds managed to assemble the picture only with help, i.e. they used a master image. They would not know what to do with individual pieces without it. Half of four-year-olds and five- and six-year-old children managed to assemble the picture within two minutes.

Three quarters of children started assembling the picture from its upper left or right corner. One quarter of children started the assembly from lower middle part. Four- to six-years-old children progressed systematically. Children at the age of three would take a piece and put it back again because they did not know where it belonged. Subsequently they would take another piece and come back to the one set aside only later on.

3.6 Task No. 5: Building a Castle from Blocks (Reproduction)

There are wooden blocks of various colours and shapes ready on the table. The child’s task is to reproduce a castle from the blocks exactly according to a model (paper plan). There is also a plane-space transformation involved when executing this task (Fig. 4).

![Figure 4. Building a castle from blocks (layout at the start).](image)

Children at the age of three did not understand the task and were unable to build according to the model. They did not abide by block colours or shape of the building. They did not finish the task with help either.

Nearly half of four-year-olds needed some help to compare their building with model and to solve the task correctly. They would swap block colours on the building. When they inspected their building against the model, they autonomously put exchanged blocks on correct positions. Children at the age of five and six built the castle according to model within three minutes.
An interesting building strategy has been noticed. A girl sorted the blocks according to shapes at first. During the actual building, she moved the pile of cuboid blocks closer to herself and built the construction base from them. She took the pile of cubic blocks, built the given part of castle and in the end she did the same with pile of blocks in the shape of triangular prisms.

3.7 Task No. 6: Comparison of Two Pictures (Natural Comparison)
The child has two pictures in front of him/her. His/her task is to find six differences (Fig. 5, Fig. 6).

![Figure 5. Comparison – differences.](image)

![Figure 6. Comparison – differences](image)

In comparison to the first task where children also confronted two pictures, this task was more difficult for them. There are two more differences between the pictures. Three- and four-year-olds struggled to remember how many differences they had already found and how many were still missing. It was necessary to count the differences for them in order for them to focus on searching. It was also necessary to remind three-year-olds of those differences that they had already found. There were no problems registered with five- and six-year-olds when solving this task.

Children chose the same strategy as they did for the first task. At first, they were attracted by different colour of one of the spires. Further they would find differences in places where there was something missing or redundant (a hill, the sun and a bush), at last they searched for differences in changed shapes (door and flag).

3.8 Task No. 7: Arrangement of Animal Pictures (Time Arrangement)
There are four pictures of animals on the table. All animals appeared in the fairy-tale previously told to the child. The child’s task is to order the pictures exactly as the animals appeared in the fairy tale (from first to last).

The picture arrangement is closely interconnected with the need to remember the tale plot that served as a motivation for single tasks.

Three-year-old children knew there were some animals in the tale but they were unable to arrange pictures in correct order, thus they did not complete the task. Four- and five-year-olds solved the task by re-telling the story together with experimenters. Six-year-olds re-told the story independently or they remembered the plot and completed the task.

3.9 Task No. 8: Arranging Cards with Pictures by Tale Plot (Time Arrangement)
There are eight cards with pictures depicting various scenes from the fairy tale. The child’s task is to order the cards according to the story unwinding, based on his/her own auditory memory.

This task was very difficult for children. The fact that small children cannot orientate in time had a major impact. Experimenters went through all the cards together with each child that needed help and
children said what was depicted on them. Then the cards were spread on the table again for children to arrange them correctly.

Three- and four-year-olds put two or three cards in front of them in the correct order but did not know how to proceed. Most five-year-olds and a fifth of six-year-olds re-told the story with experimenter’s help and then ordered the cards correctly. The rest of six-year-old children did not have any problems to solve the task.

It was discovered from results of this task that there were too many pictures for three- and four-year-olds. If there were only four cards, they would have probably solved the task.

4 CONCLUSIONS

Comparison is one of basic mathematical methods, which are the first ones to develop at children. Children solved all tasks in this area autonomously or with assistance. The results indicate that already toddler-age children start comparing things intuitively. Their experience was projected into the solution of given tasks. The biggest difficulties occurred when solving task no. 3A where they were supposed to compare weights of three purses. The youngest children were looking for visual differences and did not focus on weight comparison.

The “Whole and its parts” area is an area of average difficulty according to results. There was an obvious problem with complex perception at the youngest children. The most difficult task was finishing second half of picture. Only from the age of five on children understood the task and were able to imagine the castle both as a whole and its parts. The fact that completion was combined with axial symmetry applications played apparently a major role. There was the same problem in the perception of a whole at task no. 5, where children were asked to build a castle from blocks according to a model. All children managed excellently the assembly of a picture, younger children needed a master image.

Arrangement turned out to be the most difficult discipline. Three- and four-year olds struggled to remember the tale plot and that is why they could not solve tasks no. 7 and 8. The number of cards with the tale story also played a major role in success rate of task no. 8. Three- and four-year-olds would probably put in correct order three to four cards, however a higher number was a big problem for them.

REFERENCES


