IMPORTANCE OF LOGICAL-MATHEMATICAL ALGORITHMS IN SCHOOL MATHEMATICS

Diana Starja¹, Nikolina Nikolova²

¹Elbasan University (ALBANIA)
²Sofia University (BULGARIA)

Abstract

In our daily life we all carry out a series of actions according to a clearly defined way of solving various problems. The total of these actions is called an algorithm. Depending on the things we do to solve a specific problem, the algorithms can be logical or numeric. The first are the logical conclusions of our experiences, our actions are repeated continuously until they are imprinted into our consciousness, and we use them as readily available.

They are also the beginnings of a reasoning that, as years go on, opens the way for the creation of numerical algorithms that are widely used in school mathematics and beyond in computer science.

In this paper, we will discuss what algorithms are and how many places they occupy in the undergraduate mathematics education, how often we encounter algorithmic elements in our textbooks, ranging from simple logical ones up to the linear, branched, or cyclic ones, depending on the way actions are organized.

Keywords: logical algorithm, mathematical algorithm, linear algorithm, branched algorithm, cyclical algorithm, flowcharts, textbooks.

1 ALGORITHMS. GENERAL KNOWLEDGE.

1.1 Algorithm

In this paper, we will discuss one of the key points of computer thinking, which is an algorithm. Computational thinking plays an increasingly important role in today's education, so the sooner we introduce it to our schools the sooner we develop the analytical skills of every child.

Computer Science Teachers Association (CSTA) and The International Society for Technology in Education (ISTE) has developed a sophisticated concept of computational thinking on nine concepts and skills that are:

Computational thinking, data collection, data analysis, data representation, decomposition of problems, abstractions, algorithms and procedures, automation, parallelization and stimulation.

As we see, one of the key components of mathematical thinking is the algorithm.

In every moment of the day we are faced and we are asked to solve various problems. In some cases, we act immediately without careful reflection, such as a friend's phone call, a TV channel change, and so on.

However, when we have to deal with more complication situations or with problems that we face for the very first time, a careful analysis to choose the right path for solving the problem is inevitable.

This analysis should go through the following steps:

1. Defining initial conditions.
2. Identification of what we have to achieve.
3. Identification of the tools that we need to be available to operate.

Conducting the analysis above leads us to design the basic operations to solve the problem, building what we often call the problem-solving algorithm.
Regardless of the subject matter of a mathematical problem, it is important that planning to solve a mathematical problem should lead to a definitive end solution. By using computational thinking we can break down the problem into smaller parts and then unify the procedure to a complete solution of the problem. The list of these actions listed in the order of their performance is nothing more than an algorithm for solving a problem.

The problem-solving algorithm has the following characteristics:

1. It consists of a number of consecutive steps, fully defined, until the final solution of the problem.
2. There should be no space or ambiguity about the manner of performing and interpreting the actions.
3. It should be valid for a certain category of problems, which means through this algorithm we should be able to solve all problems of the same type.

To summarize, we must show in details all the actions we are going to do to find the solution. In other words, to solve a certain problem, whatever it is, it is required to design an algorithm. Building the algorithms is also a key point of the computational solution of that problem.

Algorithms can be built not just for solving mathematical problems, but for solving problems of problems of any type.

Regardless of its nature, an algorithm should be general and should be used as easily as possible.

The human need of responses and their will to answer questions, which has led to the creation of algorithms.

From this point of view, people prefer the answers to their questions to be correct and achieved by objective methods.

The more efficient these methods are and the more they ensure their co-operation with others methods, the more effective they are.

The algorithms should be based on unique ideas.

We should keep in mind that an algorithm created for solving a given problem should be accurate, feasible, and efficient.

After specifying the problem, a detailed description of all the actions to be carried out to reach the solution must be made.

This description is nothing more than an algorithm.

The construction of the algorithm is also the most important stage of the computational solution of a problem.

Let's give a more general definition of the algorithm.

"The algorithm is a finite set of strictly defined actions that, executed starting with initial data, yields the corresponding result and ends in a finite time."

In other words, the execution of an algorithm causes a sequential process, that is a series of actions that occur one after the other, each with a clearly defined beginning and end.

The algorithm describes solving the problem independently of any programming language.

### 1.2 Types of algorithms

By way of organizing actions, algorithms can be:

1. **Linear.**
   Actions are ranked one by one, each action is performed only once.

2. **Branched.**
   According to the value of logical conditions, a series of actions or another are performed.

3. **Cyclical.**
   One or more actions are performed repeatedly.
The description of the algorithms can be done in different ways.

Writing algorithms with a language similar to programming languages facilitates the transition from an algorithm to a program.

One way of presenting the sequences of an algorithm is graphically by flowcharts. A flowchart is a graphic representation of how a process works, showing, at a minimum, the sequence of steps.

As noted above, in different examples of algorithms the most usable logical structures are:

1.2.1 Linear structure

The simplest algorithm is the linear one, in which the actions are ranked one by one. So each of them is performed only once and simply after the preceding action has been taken. The algorithm objects (dimensions participating in the algorithm) are different. They may be constant or variable, numeric or literal, etc.

1.2.2 Branched structure

In this structure, if the logical condition is true, then the action on block 1 is performed, while the one on block 2 is passed and it is continued with the action that follows the last semicolon. If the condition is not true, then the action on block 1 is passed and the one on block 2 is performed, then it is proceeded with the subsequent action.

1.2.3 Repeated actions. Cyclical algorithms

If the solution of a problem requires execution of an action or a set of actions, then cyclic algorithms are used. Here we distinguish two cases according to how the body (repetitive actions) depend on the logical condition:

* The first case - pre-condition cycle

The logical conditions precede the body. If the logical condition is not true, then the action block that is included between the word "execute" and the last semicolon is not performed. Instead of this, it is proceeded with the next action. If the logical condition is true, then the action block is performed and the logical condition is re-evaluated.
If it has the true value again, the action block is again carried out and so on until the condition becomes false (No).

**The second case-- post-condition cycle**

In these structures, the block of actions precedes the logical condition. The action block is performed at least once, until a control is carried out.

In other structures, it is allowed to perform an instruction or block of instructions repeatedly, whereas the cyclic control variable takes progressive values.

Thus, the action or block of actions is performed for the value of the control variables from the initial value to the final value, changing in each cycle with the given step. When the value of this variable exceeds the final value, the block is no longer repeated.

---

**Figure 4. Post-condition cycle.**

---

2 **THE PRESENCE AND THE IMPORTANCE OF SIMPLE LOGIC-MATHEMATICAL ALGORITHMS (GRAPHICALLY EXPRESSED) IN PRE-UNIVERSITY SCHOOL MATHEMATICS IN ALBANIA**

School mathematics textbooks should be a good source of mathematical and logical knowledge and skills that students need to develop at the pre-university level.

*For example.*

a) Brushing teeth every morning imposes on each of us the use of a certain algorithm.

b) Passing the traffic light imposes another rule procedure to follow, which has different specifications from the teeth brushing.

Common among them is that both rely on logical actions and conclusions.
As for the mathematical knowledge that students receive, in the Albanian curricula this is designed in a spiral form, meaning that they are taken over and over again, raising the level of reasoning and knowledge gradually - year after year.

In this context it comes a moment that the logic-mathematical actions that students have received during a particular cycle are embedded in their conscience and provide them with certain math algorithms.

Since the first elementary grade, our students deal with problems and exercises that allow the usage of certain algorithms, which most students can derive from their life experiences until then.

All of these are nothing but sequential actions (each action is performed only once), so they are linear logical structures.

And there is no way this could be different, as these types of structures are even more understandable to this age compared to other structures.

First of all, in the first grade of elementary school, it is interesting to note that in almost all of the book’s exercises, the way in which requirements of exercises are dealt with, shows the student the steps that they need to perform to solve a certain exercise.

For example, on topic “Substraction of number 5”, in Mathematics 1, [2] (Diana Starja, Zenepe Shkoza, Mathematics 1 (1st grade), “AlbPAPER”, 2011) the questions of exercises are: Count how many there are. How many of them are colored? Count, how many are remained. Complete.

\[ 5 - 1 = \ldots \]
\[ \ldots - \ldots = \ldots \]
\[ \ldots - \ldots = \ldots \]
\[ \ldots - \ldots = \ldots \]
\[ \ldots - \ldots = \ldots \]

*Figure 5. Subtraction of number 5.*

The same structure is maintained in the text of the second grade as well, in all the exercise’s questions. In this grade there appear for the first time short instructions in the form of “commands”, which show the procedure followed to solving the problems.

All of these are nothing but sequential actions (each action is performed only once), so there are linear logical structures.

There is no way to do different, as these types of structures are even more accessible to this age.

In the primary school grades the most common algorithms are the linear ones, as the best for the level of reasoning of the students of this age group, but sometimes there are also simple forms of the other two types.

Below are 2 examples given to third-grade pupils to be solved in Mathematics 3 [3] (Diana Starja, Zenepe Shkoza, “Mathematics 3” (3rd grade), “AlbPAPER”, 2009). Example 1 (Fig.6)

```
*1. You see the traffic lights and you have to cross the other side of the road.
What are you going to do?
First, describe your actions and then complete the scheme.
1. Look
2. If it is green,
3. If it is not green,
```

```
I look…………………………….  
Yes

Is it green?  
No

……………………………
……………………………

*Figure 6. Example 1, 3rd grade*
Example 2. (Fig. 7)

Thus, in the fourth elementary grade, a student can list the actions he/she must perform in order to write the sum of the same summands as a multiplication and proceeds according to a certain rule to find the value of an arithmetic expression without or with brackets.

In this grade, the steps that are followed for solving problems are described in detail.

On topic: "Problem-solving" in Mathematics 4.[4](Diana Starja, Zenepe Shkoza, “Mathematics 4” (4th grade), “albPAPER”, 2007) is given to students to solve this exercise:

Example

Below is given the rules you need to follow to solve a problem
Put them in the order you need

- Determine what is required by the problem.
- Read the problem carefully to understand it fully.
- Find out the data you need about the problem situation to solve it.
- Select the action you need to solve the problem.
- Solve the problem.

It is important to emphasize that in the first grades of elementary school, even though a simple language is used to introduce the scientific material, its precision is maintained. Thus, in the fourth grade, the first elements of the specific presentation of the operations performed in an algorithm (flowcharts) are introduced to the students.

In this way alongside the mathematical, science material, the first steps of learning to use a simple computer language are taken by the students.

The variety of examples associated with graphic illustrations stimulates students of this age to consider the situation thoroughly and to cultivate the first elements of critical and analytical thinking.

In the fifth and sixth grade, the examples begin to be more and more mathematically oriented and present different mathematical algorithms.

In sixth grade Mathematics 6 [5] (Diana Starja, Zenepe Shkoza, “Mathematics 6” (6th grade), “albPAPER”, 2008) conditioned structures start to get used, i.e. finding the median in a list of numerical data.

(Here different possibilities have to be taken in consideration, such as if the number of numbers is odd or even.)
Up until the seventh grade cyclic structures with mathematical character are rarely found, because they are more complex to be structured.

Students are already familiar with linear logical structures and use them easily. The material in the text also favors this fact. For example, seventh grade students easily present statistical data in a cyclic flowchart the steps of a linear mathematical algorithm.

In the eighth and ninth grade, in parallel with the development of the ability to reason abstractly, the ability to communicate mathematically has been developed.

In this stage, reasoning becomes more and more organized. The specific weight of deductive treatments compared to inductive ones during these grades increases. Since the logical learning character is gradually strengthened, the place of linear logical structures treated in the lower grades is taken by the conditioned and cyclic structured. Of course, the latter are still not very common.

I use the phrase “the latter are still not very common”, because many topics, where the methodical treatment of the new material could be made by the flowcharts (such as “Solving the second power equation with one variable” “Mathematics 8” [6] (Diana Starja, ZenepeShkoza, “Mathematics 8” (8th grade),“albPAPER”,2012) using a cyclic structure with mathematical character, is in fact provided by the successive instructions to be followed in order to solve it. Methodological treatment of a lesson aiming at creating an algorithm for solving a given problem causes students to prioritize the initial conditions of the problem being put forward for the solution and makes them rational in structuring the solution and finalizing it.

This also represents the value of such early treatment of algorithms in school math, as it is the task of the school to prepare students to think more clearly about the concepts of mathematics and science in general.

Regardless of the algorithm we choose to solve a certain problem, the very important fact which needs to be understood by all, especially by teachers, is that the math is a mental activity writing on it textbooks is just a sort of help.

3 CONCLUSION

The graphical methods of presenting algorithms (flowcharts) that are dealt with in school mathematics are understandably usable for relatively simple problems that require a limited number of steps, but this method turns out to be difficult to handle, when problems become more complex.

However it is the most common way of representing an algorithm, as it is the simplest way to use and to be understood by someone who faces these arguments for the first time.
Graphic presentations (flowcharts) are not the only tool for algorithm presentation but this method turns out to be difficult to handle, when problems become more complex.

Showing that starting from early grades and graphical descriptions of daily life algorithms, the students go through math and toward computer programming, developing in parallel their critical and analytical thinking.

Algorithms are on the base of the development programs. The algorithms are very old and they are developed to solve mathematical and commercial problems. Nowadays the usually linked to the programming, but actually we use them everywhere (as in the examples).

Used early in school mathematics, they create the basis for tomorrow's computer programmers.

ACKNOWLEDGEMENTS

The research is partially supported by the Sofia University “St. Kliment Ohridski” research science fund project N 80-10-210/17.04.2019 Organizational models for extracurricular activities in the context of recent curricula in the cultural and educational area “Mathematics, Informatics and Information Technologies”.

REFERENCES