DEVELOPING MATHEMATICAL THINKING THROUGH EDUCATIONAL ACTIVITIES ON THE GEOBOARD

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Abstract

Mathematics is an important part of the educational system. Learning mathematics is necessary for an individual´s full development in today´s coplex society. However, we often encounter negative attitudes of pupils towards mathematics. Pupils often refer to not understanding mathematics, they consider that mathematics is too difficult for them. We believe that the beginning of these negative attitudes arises already at primary school. Pupils learn often mathematical subject matter without deeper understanding. Emphasis is placed on memorising and knowledge of algorithms. That is why we are interested in non-traditional learning enviroments which can develop pupils´ mathematical thinking. It means such educational enviroments which can contribute to changing negative pupils´ attitudes towards mathematics.

The paper sumarizes the partial results obtained in educational experiment Geoboard in Primary Mathematical Education (GPME research) which was conducted with 3rd grade pupils of primary school. This experiment was focused on developing mathematical thinking through educational activities on the geoboard. The main research methods used were participating observation and semi-structured interview. The research was of a qualitative type. The level of pupils performance in mathematics and their gender were chosen as the independent variables. The success rate of the task solution was the depend variable.

All activities were accepted by the pupils with enthusiasmus. Pupils required additional activiries beond the experiment. The pupils themselves invented new tasks. Weaker pupils needed more time but they did all the tasks successfully. Based on the results of the educational experiment, we assume that geoboard activities are a suitable activation complement to primary mathematics lessons.

Keywords: mathematics, learning, geoboard, primary school.

1 INTRODUCTION

Mathematics is an important part of the educational system. Learning mathematics is necessary for an individual´s full development in today´s coplex society. However, we often encounter negative attitudes of pupils towards mathematics. Pupils often refer to not understanding mathematics, they consider that mathematics is too difficult for them. We believe that the beginning of these negative attitudes arises already at primary school. Pupils learn often mathematical subject matter without deeper understanding. Emphasis is placed on memorising and knowledge of algorithms. [1] That is why we are interested in non-traditional learning enviroments which can develop pupils´ mathematical thinking. It means such educational enviroments which can contribute to changing negative pupils´ attitudes towards mathematics.

We were interested in further possibilities of how to familiarise pupils with mathematics with understanding. Our attention was drawn by the environment of a geoboard in specialised publications [2], [3], [4]. The geoboard is not frequently used in school mathematics in the Czech Republic. The aim of our communication is to point to geoboard activities as one of the methods that helps pupils at the 1st level of elementary school to better understand mathematical concepts and relations between them.

2 GEOBOARD

2.1 What is geoboard

A geoboard is a didactic aid used as a manipulation tool for teaching elementary geometry and other mathematical topics introduced at elementary school. It is a board covered with a grid of pins (nails) on which one can place different-coloured elastic bands and thus form vectors and polygons. Scandret [4] states that, traditionally, this aid is made of wood; however, today it is mostly made of plastic. Carroll [5]

lists areas of mathematics where geoboards can be used. These include planar formations, axial and central symmetry, rotation, similarity, arithmetic, model and image. Geoboards are also perfectly suited for deducing the circumference and area of mathematical formations, learning angles, the identity of geometrical formations, fractions and others.

The geoboard and its variations were invented and made by the Egyptian mathematician and teacher Caleb Gattegno (1911-1988) in 1952. He used it in dynamic approaches to teaching geometry. In addition to working with the geoboard, he was also concerned with innovative approaches to teaching mathematics. He did not agree with memorising facts and algorithms. In his theory, he advocated that mathematisation was not only about the pleasure of discovery but also about the associated ability of effective memorisation. [3]

Gattegno [2] presents the benefits of working with the geoboard and says that, when children work with elastic bands on geoboards, they are often capable of discovering numerous relations existing in the given situation, and are thus able to better memorise the fact or procedure in question. The method lies in that the pupils are first given time to examine the geoboard, followed by gradating tasks, thereby creating situations that lead to gaining new knowledge.

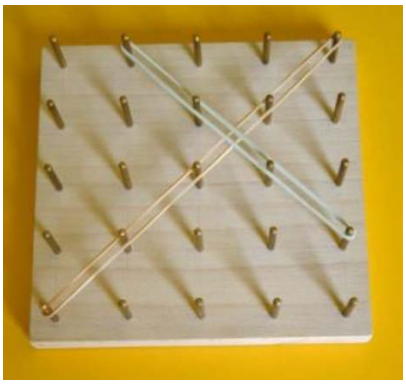

Figure 1 Geoboard 5 x 5

Žilková [3] provides a classification of geoboards according to the pin arrangement and divides them into three groups:

- **Square** - the pins are arranged in the shape of a square grid. There are different types of these geoboards according to the number of pins used, such as a geoboard *3 x 3* with nine pins, *4 x 4* with sixteen pins, and larger.
- **Triangular** - the pins are arranged so as to form a “triangular grid”. As a rule, an equilateral triangle is used.
- **Circular** - the pins are arranged in regular intervals into the shape of a circle and a pin may also be placed in the centre of the circle.

2.2 Why can geoboard be used in teaching?

By working with this aid, we can fulfil numerous focuses of the area of education:

- the use of mathematical knowledge and skills in practical activities,
- perfecting logical thinking in critical judgement,
- learning to use basic mathematical concepts and relations and being able to recognise the characteristic properties of formations,
- trying mathematical modelling, its evaluation and identifying the boundaries of its use,
- planning problem solving and selecting the correct procedure,
- developing graphical expression (using record sheets),
- supporting the development of cooperation and belief in one’s own skills in solving problematic tasks in order to realise that the task result can be reached in more ways than just one.

3 GEOBOARD IN PRIMARY MATHEMATICAL EDUCATION (GPME research)

The paper sumarizes the partial results obtained in the educational experiment GPME *(Geoboard in Primary Mathematical Education)* which was conducted with 3rd grade pupils of primary school from May to June 2016. This experiment focused on developing mathematical thinking through educational activities on the geoboard.

The main aim of the GPME research was:

- to verify the motivational character of work with the geoboard, effectivity of the tasks and used educational aids in practice,
- to compare the performance and approach of pupils to the work with geoboards and to verify the ability of pupils to redraw the results from the geoboard to recording sheets.

The main research methods used were participating observation and semi-structured interview. The research was of a qualitative type. The level of pupil's performance in mathematics and their gender were chosen as the independent variables. The success rate of the task solution was the depend variable. The experiment was based on five thematic lessons.

The research included respondents' anamnesis, a description of the observation of work in individual lessons, a transcript of non-standardized interviews conducted with pupils during the study period, a set of respondents´ recording sheets. The mathematical content involved following: a line segment, a line segment lenght, a quadriteral, a quadriteral area, a quadriteral perimetr, a triangle, a triangle area, a triangle perimeter.

There were three main research questions:

- Do the geoboard activities have a motivational character for pupils?
- Do the geoboard activities contribute to pupils´ divergent thinking?
- Can weaker pupils master cognitive-intensive tasks in the geoboard enviroment?

For the purpose of the research, a set of geoboard activities was compiled. We were inspired by the works of T. Scava [6] and K. Klhufkova [7]. The activities were divided into five thematic teaching units. All activities were directed to the area of geometry. The first unit (in research was labeled as 0) was aimed at informing respondents about the geoboard environment. The second unit (in research was labeled as 1) focused on the segment line. The third unit (in research was labeled as 3) contained activities related to the modeling of squares, calculating their circumference and content. The fourth one (in research was labeled as 3) developed the theme of rectangle. The last one (in research was labeled as 4) dealt with the issue of triangles. Each unit contained one motivational and at least one creative task. One of the tasks, called "verifying task" was included both in the first and final set of tasks in order to observe any improvement in the performance of the respondents. All respondents participated together in the first lesson. The following lessons were conducted on the individual bases.

3.1 Description of a representative sample of respondents

The selection of the representative sample of respondents was deliberate. The aim of the selection was to capture pupils with different relationships to mathematics, different mathematical success, different temperament and social background. The original number of respondents was six - three boys and three girls. However, one girl got sick during the investigation. We believe that the choice of respondents is essential for qualitative research. That is why this paper gives so much attention to the respondents' amnesia.

Respondent A: a 10-year-old boy is not very successful at school. He repeated the first class and his IQ has hardly reached the level of normality. He is in the care of the Pedagogical and Psychological Counseling Center (referred to as PPP). His PPP report states that his level of intellectual abilities is below average. He has poor eye-hand coordination, poor hearing perception, below average talent levels and weakened perceptual motor function. He comes from a single-parent family where she lives only with his mother.

Respondent A was chosen because the geoboard offers a great possibility to give pupils a variety of explanation of the subject matter. We wanted to verify that even pupils who do not benefit in the regular mathematics class can be successful in working with this didactic tool.

Respondent B: a 10-year-old boy is an average pupil. He enjoys mathematics if he is successful and in good mood. His drawback is a low concentration potential. He grows up in a single-parent family. His mother is very busy, his year-older sister takes care of him. He spends most of his free time with his older cousin in “the street environment”, so school duties are not important for him.

This respondent was chosen because he is hardly interested in any learning activities. The aim was to find out whether work with the geoboard could motivate such a pupil and help him to concentrate.

Respondent C: a 9-year-old boy is successful in normal mathematics lessons. He is very inquisitive, active and communicative. He comes from a divorced family where the biological father was soon after divorce replaced by his mother's new friend. So the new social situation is not so stressful for the boy and he can fully concentrate on his school and extracurricular activities.

His openness and honesty were the main reasons for choosing this respondent. Each of his reactions was straightforward and spontaneous - so it made him an ideal candidate for the research.

Respondent D: a 9-year-old girl is among the above-average pupils in mathematics. She understands the connection between phenomena very quickly and enjoys it in general. This girl comes from a complete, harmonious family, which is reflected in her performance. She works in a balanced, motivated and overall content way.

The reason for choosing this pupil was the need to check whether the work with geoboard would satisfy even above-average pupils. Developing skills and ritching their knowledges are interested in with this education tool.

Respondent E: a 9-year-old girl prefers sports to mathematics but she is very diligent. Her mathematical performance are average. She is not very bright but she compensates it with hard work at home. Although she does not understand everything right in school lessons she tries to catch up it through her homework. The pupil comes from an incomplete family. The relationships between parents is good but the father lives far away, so it is a reason that the girl does not often meet him. Since the mother is very busy, the girl is often at home with her five-year-old brother and take care of him.

This pupil was chosen because she is a representant of a large number of pupils with a similar “cold” attitudes to mathematics. The aim was to find out if the geoboard would increase her interest in mathematics or not. Another reason for the selection was the possibility to compare the average boy-pupil and the average girl-pupil.

Respondent F: a 10-year-old girl comes from a non-stimulating family environment. She has two siblings and spends with them all days in the garden. She is unconcentrated in lessons and therefore does not excel in mathematics, she does not enjoy.

The selection of this pupil was conditioned by the same criteria as the selection of respondent A. We would like to compare of a boy-pupil and a girl-pupil from similar social enviroments.

3.2 Description of the research investigation

The initial meeting was held with all respondents together. Our intention was for not to make respondents to feel stressed during the first meeting and new research activities. On the basis base of observational approaches, the presence of their classmates provided them with the necessary confidence and support in their new activities. All other meetings (classes) were on the individual base. Pupils worked independently under the direction of the researcher. They performed the set of tasks under the researcher leadership at each meeting. This paper does not cover a detailed description of all pupils´ activities and their analysis. In the text we will focus especially on the success of solution of the "veryfying task", which was included both in the first and final set of tasks. We will describe some important problems or unexpected procedures in solving selected activities.

3.2.1 First entry meeting - lesson zero

The aim of the first meeting was to introduce students to the geoboard and how to work in the geoboard educational environment. None of the respondents had ever met geoboard yet. During the first meeting with the geoboard without rubber bands the respondents suggested: “It is a fakir´s bed” (respondent A), "this is a key holder" (respondent F), “you can build something there” (respondent A). The others had no suggestions.

When respondents received colored rubber bands, they immediately began to try to use the rubber bands. Than they were asked to create a joint free creation separately by boys and by girls. The results

of the initial open activity are shown in figures 2a and 2b. Interestingly, the girls chose the abstract pattern and the boys the real pattern of the building.

Figure 2a, 2b. Solution of a task T: 0,2 by a group of girls (2a) and a group of boys (2b)

At the first meeting the research included the “entry-verifying” task. T: 0.3. *Identify the area of the shape on the geoboard*. As expected, none of the respondents solved the task correctly. We have noted the answers: *I do not know* (respondents A, B did not solve the task), the other respondents (respondents C, D, E, F) try to solve it but uncessesfully. The graphical input of the task T: 0.3 is shown in figure 3a and 3b.

Figure 3a, 3b. Solution of an entry task T: 1.0 (by respondets A and C)

3.2.2 Lesson 1 - Topic: line segments

The respondents did not have any major problems with the modeling of the line segments. We were surprised by their creativity in solving tasks. As the key task we consider the task No. T: 2.3: *Find out which is the second shortest line segment on the geoboard and model it.* The solution showed that pupils prefer horizontal and vertical perception of the geoboard work enviroment. Only respondent E was able to solve this task immediately, without help, although she first created a wrong 2-unit line segment. But she immediately took it off and said "This is going to be some catch, right? There must be another line segment!” She took a strip of paper that was avalable and began to look for the second shortest line segment. With this educational tool she found it very quickly. The other respondents modeled a 2-unit line as their solution. They had to be offered a strip of paper by researcher to verify the correctness of their solutions. They found the right solution only after the interview.

The last task was creative. T: 1.10 *Can we create more than 10 lines from one point?* Check on the geoboard. Selected solutions of respondents are presented in Figures 4a-4d.

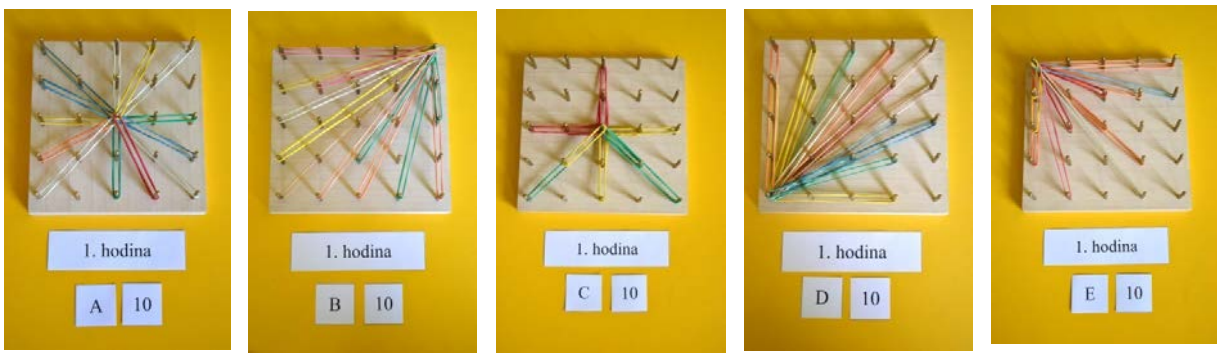

Figure 4a, 4b, 4c, 4d, 4e Solution of task T: 1.10 (by respondets A, B, C, D. E)

3.2.3 Lesson 2 - Topic: Squares

The modeling of any squares on the geoboard was for all respondents relatively successful. The key task was T: 2.3,: *Create a square with the second smallest area (less than 4 square-units).* Only respondent C completed the task successfully. First he modeled the smallest square on the geoboard. Next to it he modeled another one with the area of 4 square-units to see the difference between them. He modeled the second smallest square into the larger square. Thus, the resulting square that had an area larger than 1 square-unit and less than 4 square-units. He did away the remaining rubber bands and got the right solution. His solution You can see his solution in figures 5a-5b.

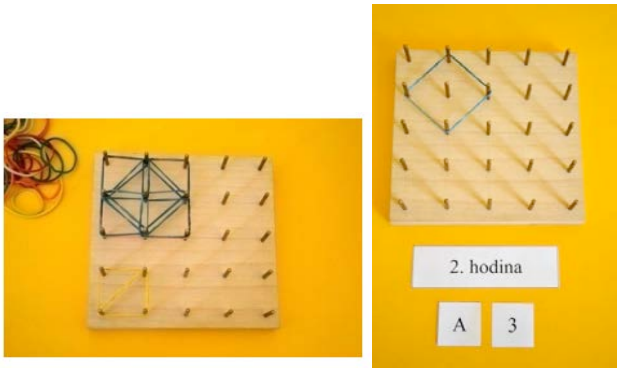

Figure 5a Solution process of task T: 2.3 (by respondent C), 5b Solution of task T: 2.3

The other respondents did not solve this task correctly. They did not fully understand the difference between the terms the square area and the square perimeter. In visual perception they prefered the system of vertical and horizontal lines, they did not accept diagonals lines of the geoboard yet. It was appropriate to use a didactic tool to understanding this task. This aid was a square, with one square unit, diagonally divided into two identical triangles (figure 6). In the following, we will call it a "triangular method".

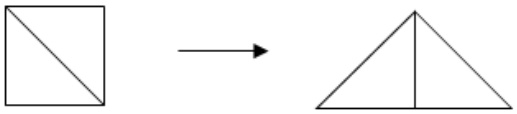

Figure 6. The principle of the triangular method (task T: 2.3)

Respondents A, D, E solved the problem right after understanding the "triangular method". Respondent B did not want to think about the solution of the problem, so we had to do it together with the help of the researcher. Respondent C did not need the aid at all.

3.2.4 Lesson 4 - Topic: Triangles

The fourth last lesson consisted of two parts. The first part was devoted to the work with triangles and in the second part the pupils were given an “output-verifying” task to calculate the pentagon area. The task was the same as the “input-verifying” task to monitor the progress of the respondents.

The “output-verifying” task was resolved by all respondents correctly.

At the beginning of the research, respondent A did not know at all how to find out the area of the shape. However, for the “output-verifying” task, he used his knowledge gained throughout the research. First he counted the squares that he had marked with the elastic bands inside the shape. Then he took the paper triangles and also to put them inside the shape. In the second step he had a problem with the correct rotation of the device, so he needed little help him. The content itself was however counted separately and successfully. His solution is in figure 9.

Responder B did not even try to solve out the “input-verifying” task but he solved the “output-verifying” task on his own, without any problems. He immediately remembered the "triangular method" and counted the content successfully.

Figure 9 Solution of the “output-verifying” task (Respondent A)

Respondent C tried to solve the “input-verifying” task by counting only the squares and therefore his wrong answer was 7. When he started to solve the “output-verifying” task he repeated this wrong solution but then he corrected himself and said, "Yeah, we have to count the remaining triangles, right?" So he filled with paper triangles two triangles and count the rest without aids.

The respondent D counted 7 square-units for the “input-verifying” task, as she did not know how to work with the remaining triangles. When she solved the “output-verifying” task, she worked by his own. She did not need the paper triangles. First, she marked all the square units, then the small triangles, and counted the area. (She counted aloud two small triangles as one square unit.) The researcher asked: "Why did you count this big triangle as one square unit?" "Because it's like two halves and they make one together."

The “input-verifying” task was conted by respondent E with a result of 6 square units. At the “output-verifying” task she counted the whole squares and did not know how to count the remaining triangles. But when she got the paper triangles she said, "Yeah, I know! I have to put them here!” And then she counted the area of the shape without a problem.

4 RESULTS AND DISCUSSION

The research was conducted in a relaxed, pleasant and friendly atmosphere. The pupils were very willing to cooperate. When anyone had a question, they were not afraid to ask. The respondents really enjoyed most of the activities and they were looking forward to the following lessons. Even after the work was finished, nearly everyone wanted to create their own pictures or come up with their own mathematical tasks. On the basis of these reactions, we can state that the work with the geoboard is motivating. The gradation of the tasks contributed to resolving difficult assignments. We verified this on several examples. However, the main proof was the fact that, upon the completion of the research, all the respondents were able to solve the verification task which they had been unable to deal with at the beginning of the research. The question of whether the work with the geoboard develops divergent thinking in pupils cannot be unambiguously answered on the basis of observations in this research. The research confirmed that even an integrated pupil is capable of solving the same difficult tasks using the geoboard as regular pupils. Working with the geoboard offers a great opportunity to work with the pupils individually, therefore even an integrated pupil was able, after a certain period of time and with the right didactic aids, to solve all tasks in the set of activities.

It was interesting to watch the progress of the individual respondents. A pupil who has great difficulties in standard mathematics managed to move to a far higher level compared to the start of the research thanks to working systematically with the geoboard. Another pupil, who is rarely interested in any activities in the school environment, was looking forward to working with the geoboard, and it was clear that the pupil was happy when he solved the tasks correctly. Yet another pupil, more than successful in mathematics, thanked for the opportunity to participate in the research and always surprised us with an original method of solving the tasks.

The geoboard has the potential of developing the pupil's understanding of mathematics. It promotes illustration and the discovery of new rules of law and experimenting. Thanks to the geoboard, mathematics becomes much more attractive for pupils.

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REFERENCES

[1] M. Uhlířová "Conceptual mapping as an inspiring strategy of mathematical lifelong learning." İn: *ICLEL 2017 Proceeding Book.* Sakarya: Sakarya University Faculty of Education, 2017, pp.245-249.

[2] C. Gattegno *Geoboard geometry*. 2nd ed. New York: Educational Solutions Worldwide, c1971-2009. Retrieved from: https://issuu.com/eswi/docs/1027_geoboard_geometry

[3] K. Žilková „Geometrické modelovanie s geodoskou." In: *Konštruktivizmus vo vyučovaní matematiky a budovanie geometrických predstáv* (O.Šedivý, K. vidermanová, D. Vallo, eds),. Nitra: Fakulta prirodných vied UKF, s. 31-38, 2010. Retrieved from: http://www.km.fpv.ukf.sk/upload_publikacie/20110629_93229__1.pdf

[4] H. Scandrett. *Using Geoboards in Primary Mathematics: Going…Going…Gone?*. In: Australian Primary Mathematics Classroom [online]. Adelaide: The Australian Association of Mathematics Teachers, 2008, vol. 13, no. 2, pp. 29-32: Retrieved from: https://files.eric.ed.gov/fulltext/EJ802704.pdf

[5] J. Carroll „Using the geoboard for teaching primary mathematics." In *M. Horne & M. Supple (Eds), Mathematics: Meeting the Challenge*, pp. 283–288, Brunswick: The Mathematical Association of Victoria, 2009.

[6] T. Scavo *Geoboards in the clasroom* [online]. mathforum.org, Retrieved from: http://mathforum.org/trscavo/geoboards.pdf

[7] K. Klhufkova *Netradiční motivační prostředí v primární matematice - práce s geodeskou.* Diploma thesis. Palacký University in Olomouc, 2017.