BRIDGING THE GAP BETWEEN ART AND SCIENCE: AN INTERDISCIPLINARY APPROACH TO THE TEACHING OF MUSIC AND MATHEMATICS WITH PRE-SERVICE TEACHERS

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Abstract

Music and Mathematics could be seen as separate curricular areas. However, some common concepts like harmony, symmetry or rhythm indicate a close relation between both disciplines. What will happen if music students develop mathematical analysis in order to understand the inner structure of a composition? Moreover, what will happen if mathematics students will use music as data for its calculations?

The interdisciplinary project “Música Matemática” has been realised by students of the degree in Primary Education, joining students from second and fourth year from the subjects “Music Education and Teaching” and “Mathematics and Teaching III” respectively.

“Música Matemática” is composed of two planning stages. Firstly, a stage of creation of musicograms based on songs or musical pieces chosen by the students. These musicograms are a kind of schematic score in which units of sound or rhythms are represented through drawings or geometrical symbols allowing children to accompany the listening of music. Secondly, those musicograms are interpreted as a compilation of graphical and musical information that becomes the source to design problems and mathematical activities to work statistics and data representation considering each symbol as a qualitative value of a graphical variable.

The final objective of the two planning stages is the development of a double class session for Primary Education, first for music and then mathematics, for which undergraduate students have to design the sessions and create the materials that will be used in them. Additional teaching methodologies develop in the sessions include active listening, cooperative work and problem based work.

Attitude of pre-service teachers after developing the project was assessed trough survey gathering qualitative data. The first results analysed support the benefits of combining both areas for an improved cross learning: students become aware of many relations between music and mathematics; were able to suggest many other possibilities to teach both subjects together apart from statistics; and could propose other possible subjects in which music could play a beneficial role.

Keywords: Music and teaching, Mathematics and teaching, Musicograms, Statistics, Pre-service teachers, Interdisciplinarity.

1 INTRODUCTION

1.1 Music and Maths Education in a connected world.

The benefits of music education are widely recognized in many fields [1]. They include cognitive, emotional and social development as well as a global improvement of academic performance [2]. The theory of Multiple Intelligences (MI) [3] is perhaps one of the approaches that best connects musical and mathematical learning.

According to Gardner, intelligences have a double base, biological and cultural [3]. Different types of learning crystallize in synaptic connections in different parts of the brain [4]. Culture, on the other hand, plays a crucial role in the development of each intelligence. Different societies and cultures value different types of intelligences. This leads us to conclude that, in the educational field, the teaching-learning processes and the application of methodologies based on the use of multiple intelligences can contribute to the development of the different intelligences in each child.

Among the 8 intelligences proposed by Gardner, the musical and the mathematics have an important connection with each other: the relationship between the musical activity and the acquisition of logical-

mathematical competences has been widely studied in different works [6, 7, 8, 9, 10], and evidence has been collected at all educational levels, including, of course, the Primary stage [11,12].

On the other hand, teaching of contents related to statistics, as part of the Mathematics curriculum, faces a necessary redefinition of materials and methodologies to capture the attention of students overexposed to all types of data and information through their social networks, media or the same physical space they inhabit [13]. One way to achieve this is to create meaningful experiences based on real data on topics close to students: music, social life, exercise, maps or mobile phones [14], and on the use of active learning methodologies, such as simulations, case studies or problem solving [15].

1.2 From music to mathematics: creating musicograms

The musicogram is a didactic resource created by the pedagogue Jos Wuytack to practice the active listening of classical music with children without musical knowledge. It consists of a series of drawings, graphics or icons that represent the different elements that are part of a musical piece: rhythmic structures, melodies, timbres, phrases and sections, etc. The association between the musical elements and its graphic and pictorial representation improves the understanding of musical elements of a high level of abstraction, such as the musical form, the length relationship between the musical figures, or the representation of a musical phrase composed of different melodies. On the other hand, it is also a very appropriate tool for the future teacher to practice the musical analysis in a very applied way, since part of the process of creating the musicogram implies performing a previous analysis of the different elements that constitutes the chosen piece of music, as well as selecting those elements that stand out for each piece and how to represent them.

The first phase of the development of the "Música Matemática" project consisted of the elaboration of different musicograms, from which to work the statistical contents. For this, each student of the subject Music Education and Teaching (ME & T) chose a musical theme, from which he/she created his/her own musicogram. With this previous work as a base, different musical elements that could be treated unitarily were identified in each song, at different levels: rhythmic cells, melodic motifs, complete musical phrases, timbral changes, parts within the musical form, etc.

Once created, each student of the subject ME & T worked in coordination with a team of students of the subject Mathematics and Teaching III (M & T III), explaining them how to understand the musicogram and defining the elements that, within it, could be used more easily in a class of 4^{th} and 5^{th} of Primary Education.

1.3 From mathematics to music: data representation and analysis

From a mathematical point of view, a musicogram can be seen as a set of visual data in which the qualitative variable “icon of the musicogram” acquires different values, being these each one of the different symbols or icons. This point of view is interesting for learners as it avoid the direct mental connection between statistical data and numbers. For sure, numbers are the basis of all sort of information but statistical analysis in the classroom should not be limited to use them, what is a common habit, but include also other elements, which are closer to students’ life, like songs, images or even stories [14].

The total number of different values and the number of times that each one of them appears (frequency), aspects related with the duration and variability of the musical works, were the first elements to take into account when adapting the musicograms to work statistics in Primary.

In the second stage of the project students of the subject M & T III, organized in groups of four people, designed activities of representation and analysis of statistical data addressed for the second and third stage of Primary Education (8 to 9 and 10 to 11 years old students). Such activities develop different strategies and contents, following the methodological approach of the work of M. A. Canals [16], namely: work with data sets and block diagram representations and promotion of actions like observation and collection of data, analysis and communication of information represented graphically and intuitive initiation to the concepts of mode and range.

The final goal of the project is completed in this second stage achieving a new knowledge about the pieces that have been listened working mathematically the representation, classification and quantification of their musical basic components. On the other hand, statistical concepts are assimilated in a warm and playful class environment.

2 METHODOLOGY

The project and the materials created by the students were tested in two experiences with three classes of Primary from the School “Marie Curie” in October 2017 and three classes of Primary from the School “San Fernando Maristas” in October 2018 both of them in Sevilla (Spain).

The students of the subjects ME & T and M & T III, from 4th and 2nd year of the Degree in Primary Education of University Loyola Andalucía (Spain) formed mixed groups, approximately of five students each, in order to attend each classroom. The professors and the tutors of Primary presented the students to the class and provided initial guidance, and then left the pre-service in charge of the session and acquire only observer status.

Two sessions of 45 minutes were realized, the first, dealing with the musical work, was conducted by ME & T students and the second, oriented to the learning of statistics, by M & T III students.

In the first part, the musical pieces were presented through methodologies of active listening that were supported by the musicograms (Figure 1). Exercises with cognitive goals, like body percussion or melodic and rhythmic movements, were combined with emotional goals, linking each music with previous experiences and emotions. In the second part, the mathematical activities were carried out by students using methodologies of cooperative work and focusing in communication and in-class debates (Figure 2).

Figure 1. Group of students using one of the musicograms during the active listening activity.

Figure 2: group of students carrying out mathematical activities using the icons from the musicogram.

After the implementation in the schools, pre-service teachers were asked to reflect about the session, describing their experiences, in all the possible dimensions: with the children, the tutors of Primary and their colleagues from the other course of the university, and then suggesting possible solutions or improvements if the session will be repeated.

Finally, after their participation in the project a questionnaire was provided to pre-service teachers in order to identify their beliefs about teaching music and mathematics in an integrated way. The questionnaire, adapted from An et al. [17, 18], consisted in three open-ended questions: Q1. What is the relation between music and maths? Q2. What mathematical concepts or contents can be studied through music? Q3. What other subjects, apart from mathematics, can be connected to music?

Answers to these open-ended questions from 33 pre-service teachers participating in teaching of music and mathematics during the years 2017-2018 and 2018-2019 have been coded, categorized and compared. The results of such analysis are presented below.

3 RESULTS: PRE-SERVICE TEACHERS' BELIEFS TOWARDS TEACHING MUSIC AND MATHEMATICS

Attitude of pre-service teachers after developing the project was assessed trough survey gathering qualitative data from the three questions asked (Table 1).

Table 1. Pre-service teachers' answers to the three open-ended questions after participation.

Q1. What is the relation between music and maths?	*nº* / *%*	*Q2. What mathematical concepts or contents can be studied through music?*	*nº* / *%*	*Q2. Number of different mathematical concepts given by the students:*	*nº* / *%*	*Q3. What other subjects, apart from mathematics, can be connected to music?*	*nº* / *%*	*Q3. Number of different subjects, apart from mathematics, given by the students:*	*nº* / *%*
Rel. between structure and rhythm with numbers (counting, rational numbers…)	12 / 34'3 %	Counting.	6 / 8'6 %	Student was able to provide five concepts.	1 / 3'0 %	Physical Education.	10 / 12'0 %	Student was able to provide six subjects.	1 / 3'0 %
Rel. between time and length with measurement.	6 / 17'2 %	Calculations.	16 / 22'9 %	Student was able to provide four concepts.	2 / 6'1 %	Arts.	15 / 18'1 %	Student was able to provide five subjects.	2 / 6'1 %
Rel. between graphical representations and language of music and mathematics.	5 / 14'3 %	Measurements: time.	6 / 8'6 %	Student was able to provide three concepts.	10 / 30'3 %	Languages.	18 / 21'7 %	Student was able to provide four subjects.	5 / 15'2 %
Rel. between instruments and geometry.	1 / 2'9 %	Geometry.	4 / 5'7 %	Student was able to provide two of concepts.	7 / 21'2 %	Literature.	3 / 3'6 %	Student was able to provide three subjects.	7 / 21'2 %
No rel. are indicated. However, their existence is accepted.	6 / 17'2 %	Statistics and data representation.	13 / 18'6 %	Student was able to provide one concept.	2 / 6'1 %	Social sciences: History.	13 / 15'7 %	Student was able to provide two subjects.	3 / 9'1 %
No rel. are indicated. The lack, or the difficulties to find them, are highlighted.	4 / 11'4 %	Probability.	6 / 8'6 %	Student was not able to provide any concept.	11 / 33'3 %	Natural sciences.	10 / 12'0 %	Student was able to provide one subject.	5 / 15'2 %
		Rational numbers: fractions and percentages.	5 / 7'2 %			Technology (ICT).	2 / 2'4 %	Student was not able to provide any subjects.	10 / 30'4 %
		Other types of numbers: Ordinal and Prime.	2 / 2'9 %			Drama.	1 / 1'2 %		
		No concept is specified. However, it is stated that many of them will meet the requirement.	8 / 11'4 %			No one is specified. It is stated that any or many of them.	10 / 12'0 %		
		Cannot give an answer or it is not related with the question.	3 / 4'3%						
Total number of answers.	35 / 100%	Total number of answers.	70 / 100%	Total number of students	33 / 100%	Total number of answers.	83 / 100%	Total number of students	33 / 100%

In overall terms, the general structure of the teaching innovation project and its activities were evaluated very positively by course participants. They greatly valued teaching music and mathematics in non-traditional ways. Specifically, students realized about the relations between music and

mathematics (Question 1). 34'3 % of students pointed out the relation between musical structures and rhythm with numbers, 17'2 % the relation between duration of sounds or silences and time measurement and 14'3 % the relation between mathematical and musical notation. A significant 17'2 % stated that there were many relations but did not name any. On the other hand, 11'4 % of the students did not see any relation or only focused on the difficulties when working the two subjects together.

Regarding the question 2: What mathematical concepts or contents can be studied through music? Apart from the 18'6 % of the students that provide the direct answer: statistics and data representation, a 22'9% of students named number calculations and in lower percentages other concepts or contents like counting, rational numbers, time measurement and geometry, etcetera. Additionally, this question is interesting if we think about what is the number of different mathematical concepts or contents that students were able to identified in their answers. From that point of view, almost a 40 % of the students were able to name from three to five concepts while one third of them, 33'3 %, was not able, or refused, to provide any.

Question 3 allows a similar analysis. When asked about what other subjects can be connected to music, participants' favourite answers were: Languages (21'7 %), Arts (18'1 %), History and Social Sciences in general (15'7 %) Natural Sciences (12'0 %) and Physical Education (12'0 %). Regarding the number of valid answers to question 3, almost a 45% of the participants named three or more subjects that could be worked together with music, while a 30'4 % did not gave any possible subject to do that.

We will like to show some specific examples of how students realized that there exist relation between music and mathematics (those comments were in response to open-ended question 1 "What is the relationship between music and math?"): "Music follows rhythmic-melodic patterns" (Participant 1); "Rhythm is based upon mathematical relations" (Participant 4); "…Musical notes are relate to numbers (one black, two white ...)" (Participant 7); "Music incorporates many mathematical concepts such as the duration of the notes, the compasses,…." (Participant 14); "…I discovered that it was possible to learn the fractional nature of notes values" (Participant 18).

Furthermore, students were able to recognize and apply knowledge to diverse content areas (those comments were in response to open-ended question 2 "What other mathematical concepts can be studied through music?"): "Statistics, probability and operations (addition, subtraction, multiplication and division)" (Participant 2); "Length, basic arithmetic or geometry" (Participant 16); "Through music you can work different mathematical concepts such as numbers, statistics, probability and measurements. Despite these concepts are complex, it is possible to integrate a variety of music activities with different mathematical content" (Participant 19). Not only pre-service teachers' beliefs but also the literature highlights how mathematics and music relate to each other. For instance, Beer [19] suggested that notes, intervals, scales, harmony, tuning, and temperaments can be relate to different mathematical concepts such as proportions and numerical relations, integers, logarithms and arithmetical operations, trigonometry, and geometry. More recently, the study of Cranmore and Tunks [20] demonstrated that musical ability and mathematics performance are related.

In addition, students proposed other possible subjects in which music could play a beneficial role (those comments were in response to open-ended question 3 "What other subjects, apart from mathematics, can be connected to music?"): "Spanish language and Literature and art" (Participant 3); "I think that all subjects can be connected to music" (Participant 10); "Spanish language and Literature, English language or some foreign languages, Social Sciences, Natural Sciences" (Participant 15). In general terms, there were a consensus in this open-ended question. Authors of this paper hypothesize that students were aligned in their responses due to the university under study is used to promoting and rewarding initiatives in which two different subjects are closely related to each other.

4 CONCLUSIONS

This research study has shown the interdisciplinary project "Música Matemática" realized between the subjects Music Education and Teaching and Mathematics and Teaching III from the Degree in Primary Education from Universidad Loyola Andalucía.

The benefits of combining the teaching and learning of music and mathematics through the use of the musicograms an active listening and the design of statistic and data representation problems has been highlighted.

Finally, qualitative data provided by students in the last three questions of the questionnaire (open-ended questions in which students could express their opinions after their learning experience) was presented. The preliminary results supported the feasibility of combining both areas music and mathematics for an improved cross learning. Specifically, pre-service teachers realized about the relation between music and mathematics (Q1); were able to recognize and apply knowledge to diverse content areas (Q2); and proposed other possible subjects in which music could play a beneficial role (Q3).

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