EDUCATIONAL SOFTWARE FOR THE INTERACTIVE STUDY OF DYNAMIC ABSORBERS

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Abstract

Elastic systems acted by periodical forces perform vibrations with amplitudes depending on the amplitude of the perturbing force, as well as on its frequency. If the frequency of the perturbing force is close or equal to one of the eigenfrequencies of the elastic system, the oscillation amplitude increases significantly. This phenomenon, called resonance, is particularly important in mechanical engineering, as it can have a negative influence the functioning of the system, may have hazardous effects on the human operator and, in extreme cases, can lead to the deterioration and destruction of the system. A relatively simple method, from the constructive point of view, for avoiding the resonance effect, consists in the use of dynamic absorbers. These represent elastic subsystems, which are attached to the main system and which, if properly dimensioned, have as an effect the reduction, up to elimination, of the amplitude of the main mechanical system. The paper presents an educational software application, with graphical interface, which simulates by animation the oscillations of a system with a dynamic absorber. The students can vary the parameters of the system, such as to highlight the effect of the absorber and to determine by a trial-and-error process its optimal characteristics. The software, aimed for use in practical sessions for the courses of mechanical vibrations taught in engineering faculties, will be integrated in a larger virtual laboratory that the authors have developed.

Keywords: Vibration, damping, dynamic absorbers, simulator.

1 INTRODUCTION

Resonance is a phenomenon that occurs in oscillating systems when the frequency of the perturbing force is close or equal to one of the eigenfrequencies of the system. In this case, the oscillation amplitude increases significantly. In mechanical and in civil engineering, resonance is avoided by specific design, as it can lead, in extreme cases, to the deterioration and even destruction of the machinery or structure in which it occurs. The amplification of oscillations caused by resonance may have hazardous effects on the health of machine operators or building occupants. Several methods have been developed, over time, to avoid resonance. A relatively simple method, from the constructive point of view, consists in the use of dynamic absorbers. These are elastic oscillating subsystems attached to the main system. If properly dimensioned, dynamic absorbers can reduce, up to elimination, the amplitude of the main mechanical system.

In engineering faculties, vibration control, i.e. the technical means that should be used to mitigate the harmful effects of oscillations - among which resonance, is taught either as a separate course or is integrated in the general vibration course. To facilitate the understanding of various vibration phenomena by the students, various mechanical laboratory experiments are performed during practical sessions, traditionally with the use didactic instruments. During the past decades, classical laboratories in various fields are, however, more and more replaced by virtual laboratories [1-9], in which the place of didactic instruments is taken by virtual simulations of the physical phenomena.

Within a larger effort of developing a virtual laboratory for vibrations and for mechanical problems in general [11-14], the authors have developed an educational software application, with graphical interface, which simulates by animation the oscillations of a system with a dynamic absorber. The students can vary the parameters of the system, such as to highlight the effect of the absorber. In addition, they can use the program to determine its optimal characteristics by a trial-and-error process. The software is aimed for use in practical sessions for the courses of mechanical vibrations taught in engineering faculties.
2 THEORETICAL BACKGROUND

2.1 The dynamic absorber

The mechanical system in Fig. 1 is considered [15, 16]. The system performs forced oscillations under the action of the harmonic perturbation force

\[ F_p = F_0 \cos \Omega t. \]  

The system consists of:

- the main system, formed by the slider of mass \( M \), coupled to the fixed element by the spring with the elasticity constant \( K \);
- the auxiliary system, formed by the slider of mass \( m \), coupled to the first slider by the spring with the elasticity constant \( k \).

The auxiliary system is called **dynamic absorber (without damping)**.

![Figure 1](image)

The generalized coordinates of the system are the absolute displacements of the two sliders, \( x_1 \) and \( x_2 \), respectively.

If the mechanical characteristics of the system are appropriately chosen, the presence of the dynamic absorber has as effect the reduction of the oscillations of the main system.

The differential equations of the system motion are:

\[
\begin{bmatrix}
M & 0 \\
0 & m
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} +
\begin{bmatrix}
K + k & -k \\
-k & k
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} =
\begin{bmatrix}
F_0 \\
0
\end{bmatrix} \cos \Omega t. 
\]  

(2)

The permanent solution of the system (2) is sought, when outside the resonance:

\[
\begin{bmatrix}
x_{p1}(t) \\
x_{p2}(t)
\end{bmatrix} =
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix} \cos \Omega t.
\]  

(3)

Bu using the notations

\[
\begin{bmatrix}
d_{11} & d_{12} \\
d_{21} & d_{22}
\end{bmatrix} =
\begin{bmatrix}
K + k - \Omega^2 M & -k \\
-k & k - \Omega^2 m
\end{bmatrix},
\]  

(4)

\[
\Delta_d = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \left( K + k - \Omega^2 M \right) \left( k - \Omega^2 m \right) - k^2.
\]  

(5)

It results

\[
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix} =
\begin{bmatrix}
d_{11} & d_{12} \\
d_{21} & d_{22}
\end{bmatrix}^{-1}
\begin{bmatrix}
F_0 \\
0
\end{bmatrix} =
\frac{1}{\Delta_d}
\begin{bmatrix}
d_{22} & -d_{12} \\
-d_{21} & d_{11}
\end{bmatrix}
\begin{bmatrix}
F_0 \\
0
\end{bmatrix} =
\frac{F_0}{\Delta_d}
\begin{bmatrix}
k - \Omega^2 m \\
k
\end{bmatrix}.
\]  

(6)

The amplitudes of the variables \( x_1 \) and \( x_2 \) are

\[
\begin{bmatrix}
|A_1| \\
|A_2|
\end{bmatrix} =
\frac{F_0}{\Delta_d}
\begin{bmatrix}
|k - \Omega^2 m| \\
k
\end{bmatrix}.
\]  

(7)
2.2 The dynamic damper

The mechanical system studied in the case of the dynamic absorber is studied again, this time taking into account the damping (Fig. 2). It is assumed also, this time, that the system performs forced oscillations under the action of the harmonic perturbation force

\[ F_p = F_0 \cos \Omega t. \]  

(8)

The slider of mass \( M \) is coupled to the fixed element also by the dashpot with the damping coefficient \( C \), while the slider of mass \( m \) is coupled to the first slider also by the dashpot with the damping coefficient \( c \).

The auxiliary system is called **dynamic damper** (**dynamic absorber with damping**).

![Figure 2](image)

Similar to the case of the dynamic absorber, the presence of the dynamic damper has the effect of reducing the oscillations of the main system.

The differential equations of motion are:

\[
\begin{bmatrix}
M & 0 \\
0 & m
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} +
\begin{bmatrix}
C + c & -c \\
-c & c
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
K + k & -k \\
-k & k
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} =
\begin{bmatrix}
F_0 \\
0
\end{bmatrix} \cos \Omega t.
\]

(9)

The permanent solution of the system (9) is sought in the form:

\[
\begin{bmatrix}
x_{21}(t) \\
x_{22}(t)
\end{bmatrix} =
\begin{bmatrix}
A_{c1} \\
A_{c2}
\end{bmatrix} \cos \Omega t +
\begin{bmatrix}
A_{s1} \\
A_{s2}
\end{bmatrix} \sin \Omega t.
\]

(10)

The following notations are used

\[
\begin{bmatrix}
d_{11} & d_{12} \\
d_{21} & d_{22}
\end{bmatrix} =
\begin{bmatrix}
K + k - \Omega^2 M & -k \\
-k & k - \Omega^2 m
\end{bmatrix},
\]

(11)

\[
\begin{bmatrix}
e_{11} & e_{12} \\
e_{21} & e_{22}
\end{bmatrix} =
\begin{bmatrix}
C + c & -c \\
-c & c
\end{bmatrix},
\]

(2.12)

\[
\begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix} =
\begin{bmatrix}
d_{11} & d_{12} \\
d_{21} & d_{22}
\end{bmatrix} +
\begin{bmatrix}
e_{11} & e_{12} \\
e_{21} & e_{22}
\end{bmatrix} =
\begin{bmatrix}
K + k - \Omega^2 M + i\Omega (C + c) & -k - i\Omega c \\
-k - i\Omega c & k - \Omega^2 m + i\Omega c
\end{bmatrix},
\]

(13)

\[
\Delta_R = \begin{bmatrix}
h_{11} \\
h_{21}
\end{bmatrix} = \Delta_R + i\Delta_I,
\]

(14)

where

\[
\begin{bmatrix}
\Delta_R = (K + k - \Omega^2 M)(k - \Omega^2 m) - k^2 - \Omega^2 Cc \\
\Delta_I = \Omega [K - \Omega^2 (M + m)c + (k - \Omega^2 m)C]
\end{bmatrix}.
\]

(15)

Also, the following notations are made:
\[
\begin{align*}
    f_{1R} &= d_{22}F_0 = (k - \Omega^2 m)F_0 \\
    f_{1I} &= e_{22}F_0 = \Omega c F_0 \\
    f_{2R} &= -d_{21}F_0 = kF_0 \\
    f_{2I} &= -e_{21}F_0 = \Omega c F_0.
\end{align*}
\] (16)

It results
\[
\frac{A_{c1}}{A_{c2}} = \frac{1}{|\Delta h|^2} \begin{bmatrix} \Delta R f_{1R} + \Delta f_{1I} \\ \Delta f_{2R} + \Delta f_{2I} \end{bmatrix} = \frac{F_0}{|\Delta h|^2} \begin{bmatrix} \Delta_R(k - \Omega^2 m) + \Delta_i \Omega c \\ \Delta_R k + \Delta_i \Omega c \end{bmatrix},
\] (17)
\[
\frac{A_{s1}}{A_{s2}} = \frac{1}{|\Delta h|^2} \begin{bmatrix} \Delta f_{1R} - \Delta f_{1I} \\ \Delta f_{2R} - \Delta f_{2I} \end{bmatrix} = \frac{F_0}{|\Delta h|^2} \begin{bmatrix} \Delta_i(k - \Omega^2 m) - \Delta_R \Omega c \\ \Delta_i k - \Delta_R \Omega c \end{bmatrix}.
\] (18)

where the modulus of the determinant was introduced,
\[
|\Delta h| = \sqrt{\Delta_R^2 + \Delta_i^2}.
\] (19)

The amplitudes of the variables \(x_1\) and \(x_2\) are
\[
\frac{A_1}{A_2} = \frac{\sqrt{A_{c1}^2 + A_{s1}^2}}{\sqrt{A_{c2}^2 + A_{s2}^2}} = \frac{F_0}{|\Delta h|} \left( \frac{\sqrt{k - \Omega^2 m}^2 + (\Omega c)^2}{\sqrt{k^2 + (\Omega c)^2}} \right).
\] (20)

3 PROGRAM DESCRIPTION

A computer program, called “Damper” was developed by the authors, to illustrate the use of dynamic absorbers and dynamic dampers for vibration attenuation.

The program main window is shown in Fig. 3. The left pane displays the main system (at the bottom) and the auxiliary one (at the top). This representation is used for showing the animation of system motion. The red arrow is the harmonic perturbation force. The right upper pane shows in real-time the variation of the displacements of the two masses. At the bottom of this pane, there are two sets of text boxes: the lower set allows specifying the input data, including the initial conditions, expressed in displacements and velocities, for the two systems \((x_{10}, v_{10}, x_{20}, v_{20})\). An additional button allows adjusting the zoom level of the system representation. The upper set, which is read-only, displays instantaneous values of motion parameters as well as the oscillation amplitudes, \(A_1\) and \(A_2\), for both systems.

The required input data are the masses \((M, m)\), damping ratios \((C, c)\) and elasticity constants \((K, k)\) – with capitals used for the main system, the amplitude of the perturbation force \((F_0)\), its circular frequency \((\Omega)\), as well as the initial conditions mentioned above.

Once the input data is entered, the user presses the button “Compute” to launch vibration analysis. To control the animations, the toggle buttons “Start/Stop” and “Pause/Continue” are used.

4 USE OF THE “DAMPER” PROGRAM DURING PRACTICAL SESSIONS

The program “Damper” can be used for the practical sessions of the vibration course taught in engineering faculties, as well as for any educational demonstration implying the use and the calibration of dynamic absorbers and dynamic dampers. The lesson integrating the use of the program has the following objectives: understanding the general principle of dynamic absorbers and dampers, observing the effect of the auxiliary system, separately for undamped and damped vibrations, for resonance (i.e. for the case in which the circular eigenfrequency of the main system is equal to the circular frequency of the perturbation force) and for not equal frequencies and, finally, the observation of the transitory and stationary vibrations, for various levels of damping.

The students are first instructed in the general use of the program (parameter signification, data input, visualisation of results etc.). Then, the first application is made, to illustrate the undamped vibrations of
the absorber in the non-resonant case. The program window displaying the stationary vibrations for this type of application is shown in Fig. 3.

Figure 3. Undamped, non-resonant, not tuned, steady vibrations

The second application, illustrated in Fig. 4, consists in modifying the circular frequency of the perturbation force $F_P$ to a value equal to the circular eigenfrequency of the main system, in order to achieve resonance. The students can observe, in this case, the motion of the two systems.

Figure 4. Undamped, resonant, not tuned, steady vibrations

The third application, illustrated in Fig. 5, consists in modifying the parameters of the previously defined auxiliary system (mass $m$ and stiffness constant $k$) in order to achieve the same circular frequency as that of the main system. This means tuning the absorber. At the same time, they are instructed to modify the circular frequency of the perturbation force, $F_P$, to a value not equal to the circular eigenfrequency of the two systems.
The fourth application is illustrated in Fig. 6. Now, the students perform the tuning of the auxiliary system, as they already know, by adjusting the parameters of the system in the previous application. They observe that the auxiliary system (the dynamic absorber) has achieved its role, as the main system has ceased vibrating (the amplitude $A_1$ of its vibrations is null).

The next two applications are performed for a damped auxiliary system, i.e. for a dynamic damper. A damping coefficient of 0.5 is used.

First, the students tune the damper and also change the frequency of the perturbation force, in order to achieve resonance (Fig. 7). They observe the transient and steady vibrations, as well as the oscillation amplitudes $A_1$ and $A_2$. 
Then, the non-resonant situation is considered, but with a tuned damper. The transitory and stationary vibrations of the system are observable (Fig. 8).

As a final conclusion, the difference between the dynamic absorber and the dynamic damper is highlighted by the instructor, who underlines the lower efficiency of the damper at resonance, but the higher efficiency, as compared with the absorber, in the range of frequency close to resonance.

The program allows also the demonstration of other situations, resulting from various combinations of input parameters.
5 CONCLUSIONS

A computer program for the simulation of dynamic absorbers was presented, aiming to facilitate teaching of vibration control problems to students in engineering faculties. The program, developed by the authors within a larger effort of creating a virtual laboratory for the study of vibrations in engineering faculties, provides the possibility of visualising an oscillating system to which a dynamic absorber was attached in order to reduce the oscillations amplitude, avoiding thus the harmful effects of undesirable displacements due to resonance. The parameters of the systems can be adjusted interactively by the students, which can observe in real-time, on the screen, the influence of input values to the oscillations, and compare it to the results obtained by analytical methods. During the practical sessions applying the program, the students are instructed to determine, by using a trial-and-error approach, the optimal characteristics of the absorber, being also given indications about the usual strategies used in the design and calibration of such types of devices. This substantially facilitates their ability to understand the relation between the analytical apparatus taught at the course and its practical correspondence.

REFERENCES

