Abstract
A new perspective in the approach to mathematical teaching in tandem with technology development has led to content integration of lineal algebra with mathematic entities. This integration results in the iteration of an elemental geometric process that originates a final structure of complex characteristics called fractals. This work describes an experience carried out in the computer lab of the University Basic Science Department with a class of chemical engineering students in the Algebra & GA subject. Students were asked to use concepts from lineal transformations to design fractal structures. Affine transformations that are part of Algebra & GA subject require a certain level of abstraction during teaching. In this line, students were invited to develop a reflexive and autonomous activity while looking into the aspects of an iterative sequence of function in the lab practice with the aim of visualizing 2D and 3D geometric structures.

The possibilities available from computational system have aided the reformulation of the teaching methods and subject syllabus including a revision of students’ competences. This leads to a multidisciplinary teaching perspective in engineering careers as students can learn mathematics supported by new technologies and simulation software to analyse different aspects of an engineering system that can be furthered over the applied technology cycle in the career. This approach also prompts academic development underpinned by innovative views which in turn encourage engineering students to approach new mathematical contents through appropriate applications. The transformation in the current teaching context may lead to the creation of virtual labs to provide meaningful learning situations and work with systems numerically and symbolically extending the knowledge of the subject area.

Keywords: Visualization, simulation, affine transformations, conceptualization.

1 INTRODUCTION
Nowadays, higher education is a field of steady revision and analysis, especially of those processes which aim at developing engineer students’ competence for future professional performance. In this scenario the articulation of different career plans as regards the selection of content and activities suitable to different competencies should be guided by the policy management of academic evaluation and construction.

The computational environments have changed the teaching-learning processes in engineering education leading to a reconceptualization of core ideas and enriching those processes with a multidisciplinary and integral character mainly in the way problems are selected and how they are presented and solved out [1], [2]. In this line, the A&GA Engineer Syllabus should adopt a platform of content management and related activities that can facilitate the communication among different subjects and can link them to foster the development of engineering models. The curriculum innovation should include multidisciplinary aspects, emphasizing the perspectives of the systems, as well as engineering problems, principles, practice and solutions early on into the teaching. For this it is necessary that certain teaching strategies are set up in the classroom to achieve a meaningful and functional learning [2].

The use of computing technology in mathematical teaching has changed the teaching methods and strategies. The mathematical model concept has been extended to include the representation of processes by means of virtual environments as well the development of algorithms simulation. These technologies are based on complex algorithms which, by means of the appropriate use, manage to generate astonishing objects unthinkable a few years ago. These algorithms have a strong mathematical base which allows the conceptualization of new methods to solve engineering problems.
Traditional engineering teaching in the areas of basic subjects fails to be structured to face these challenges. In order to achieve the analysis and use of complex design systems and generate models, it is convenient to appeal to the visualization. In this way the mathematic computational contributions are fundamental, both during engineering students’ education stage and during their professional lifespan [3].

The present study describes an experience in the A&GA subject where students develop an application of lineal transformations connecting them with the study of fractal geometry using specific software. Teaching the fundamentals of related transformations and how they can match the construction of geometrical entities show the actual associations between these structures and symbolic computing. Also the computation tools available today facilitate visualization which in turn promotes the conceptualization of content and the development of students’ skills.

2 METHODOLOGY

One underlying issue on teaching is how to create the conditions so that students’ knowledge schema can develop in a certain way. The key point is not whether learning should be focused on content or process but rather that it should be meaningful and functional. In fact, students need to count on previous knowledge from which they can approach the target contents so that the relations established can be complex and rich enough to increase the meaning of their learning [3]. It is advisable to help students reorganize and assimilate all previous knowledge necessary to approach the target content, i.e. in order to reach the planned learning objective successfully, suitable strategies are devised to prompt supportive learning situations.

The objects generated by repeated iteration from a transformation matrix have in its algebraic structures high possibility of abstraction and generalization. Relating these expressions with the visualization of the graphics of complex geometric shapes facilitates the competence development about the topic. In fact, mathematical education by visualization and simulation contributes to students’ responsibility and engagement in the target topics while fostering autonomy and promoting meaningful learning [3], [4]. The lessons are presented in the Computer and Multidisciplinary Laboratory of Basic Sciences by means of a teaching methodology known as theoretic-practical-technological. This methodology is considered systemic and integral and it turns the classroom into a workshop setting which makes use of computational tools and specific software.

The teaching sequence presented promotes communication among students to reach agreement on convergent criteria, discussions of different alternatives to problem solving and active participation to think independently. Teaching-learning strategies are designed to develop competences according to different learning styles to confront complex situations, give feedback and define possible behaviors in different scenarios [4]. Visualization, modeling and simulation require previously previous work of mathematical formalization, although it may also prove convenient to discriminate in each activity if the application of mathematics in the justification of the model can be done a posteriori as a verification or demonstration sages of what has been modeled or simulated. Thus, students work in an independent way, and they become craftsmen of their own learning as they develop autonomy and responsibility, being in charge of planning, structuring and elaborating the results to work out the questions set.

3 OBJETIVES

The actions that are implemented intend students to assimilate basic theoretical concepts of the analytical program as well as to develop their capacity for analysis and synthesis taking advantage of the rational and deductive nature of this discipline, highlighting in the teaching-learning process the inter and multidisciplinary character developed in Engineering careers. In this line, first the contents must be approached with solid theoretical conceptual grounds, with no need to include the rigor of courses oriented to exact sciences, so that students can immediately transfer their knowledge to specific applications and models. Here emphasis should be given to relationships with other disciplines within the engineering field [5].

The teaching-learning activities are planned to present experiences where students carry out self-managed and collaborative work which aims at their internalizing the concepts of the subject by designing an engineering situation that focus on different topics of the A & GA curriculum taking advantage of the technological resources. Activities also aim at train students in the use of specialized computer packages that allow them to develop skills to use the knowledge of A & GA to solve situations and model problems that arise during the course of their career.
It is important to highlight the need to integrate the generic competencies with the graduate profile and this involves a process not only derived from institutional, national or international policies but also from the actual action integrating generic competencies: team work, oral and written communication, information management, project-based learning and problem solving [5]. These competences, in addition to others, are the result of analyzing white papers that integrate generic to specific competencies to describe the graduation profiles for each engineering career.

The present experience is intended to convert the classroom into a classroom workshop where the student experiences learning generated by interaction techniques between the teacher as a passive subject and the student as an active subject of knowledge [6]. This line of work allows delineating a class that takes into account a learning transformation in as far as the student leaves the central place that has historically had within the classroom to occupy other spaces during the class, e.g. the space needed to interact with their peers and with the work inquiries.

The activities also aim at using a symbolic calculation tool and computer resources to connect Mathematics and real models and to develop a competency-based approach which contributes to the development of students’ basic skills.

4 MOTIVATION

In the A&GA teaching context, there is a certain level of concern about students’ low interest as regards how the subject contents are presented in the classroom and how students’ appropriation of knowledge is actually carried out and then used in advanced courses [6].

Concepts are presented with a certain level of generalization and abstraction by teacher not meeting students’ expectations. However, some students report this problem may originate not only in their poor understanding of concepts but also in the problems for practice and application [6], [7].

Motivation plays a fundamental role in the teaching-learning process and it is closely related to whether that process is functional and meaningful. For the training of engineers and other professionals in the field of engineering, it is appealing to use models of real situations and different methods offered by mathematics, since these resources are the ones that offer optimal solutions to the different models proposed, to be able to achieve abstract thinking [7]. The use of computer tools allows students to explore, infer, formulate hypothesis, justify, test their arguments and in this way, build their own knowledge independently from the teacher's intervention. These tools also allow the teacher to focus on stimulating and guiding the learning, although this new teacher's role requires greater teacher activity as it is necessary constant creativity in the presentation of situations that arise in the classes.

5 EXPERIENCE DEVELOPMENT

A theoretical-practical-technological class is designed and then carried out in the Computer Science and Multidisciplinary Laboratory of Basic Science Department. It is divided in two sessions of three hours each so that students can acquire knowledge of the applications related to linear transformations, highlighting the relationships of related transformations with the use of computational tools, and from these, the construction of geometric objects called fractals whose basic structure, fragmented or apparently irregular, is repeated in each different scales.

It is intended that students work collaboratively. For this reason, groups are invited to get in groups of maximum three members according to their free choice by affinity.

The class is planned with a didactic sequence that includes: the assembly of the work groups, a theoretical investigation of the related transformations, research and application of the theoretical developed contents, free design of a standard object, to finally realize the construction of friezes, mosaics and fractals.

5.1 Theoretical foundation

The students inquire about the content of the theoretical foundations of the subject using the bibliographic material available in the laboratory, and through the information provided by the virtual libraries, and the internet sites, generating a unique report and agreed by them that will serve as a guide to make the geometric objects.
Next, the conclusions reached by the students are described briefly:

If $A$ is an $m \times n$ matrix, and $T: R^n \rightarrow R^m$ it is an affine transformation if it has the form

$$T(x) = Ax + B \tag{1}$$

To any fixed vector $B \in R^m$. This transformation is non-lineal if $B \neq 0$. Of equation 1 the students conclude that $T(0) \neq 0$.

In the special instance that $m = n$ y $A$ is the identity matrix ($I$), so:

$$T(x) = Ix + B = x + B \tag{2}$$

In the equation 2 to these $T$ it is called translation by $B$.

The students enunciate from the vector form of equation 1, the matrix form of affine transformations in the plane, and such that for all $k, c, b_1 y b_2 \in R$, result the expression (3),

$$T: R^2 \rightarrow R^2 / \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} k \cos \alpha & -c \sin \beta \\ k \sin \alpha & c \cos \beta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{3}$$

where $0 < \alpha < 2\pi$, $0 < \beta < 2\pi$

Then students focus their attention on describing, studying and analyzing four types of related transformations in the plane that they will then apply in the construction of geometric objects from certain patterns. These transformations are: translation, dilation, reflection, and rotation [8].

Translation: A translation, or translation operator, is an affine transformation that moves each point a certain distance in the same direction. To perform the translation of a geometrical figure modifies the vector $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ in the equation 3.

In equation 2, a translation by a vector $B \neq 0$ displaces a figure by adding $B$ at all its points. An affine transformation is a linear transformation followed by a translation.

Dilation: The dilation or change of scale represents the uniform change of scale in the process of transformation that widens or reduces the geometric objects; the scale factor is the same for all directions. The scale factor in the direction of the $x$-axis is represented by the $k$ factor, and also the scale factor in the $y$-axis direction is represented by the $c$ factor.

In the case that rotations do not occur, and also if $k = c$, then it is said that the transformation is a similarity. Considering the origin of coordinates as a fixed reference point, dilation is always oriented towards the origin.

Reflection: Reflection transforms an object into its mirror image. To reflect a flat figure, a mirror is required to be a line (reflection axis or symmetry axis), while for reflections in three-dimensional space a plane (the plane of reflection or symmetry) is used as a mirror.

In equation 3, if the negative $k$ factor is the reflection occurs through the $y$-axis, while the negative $c$ factor is reflected through the $x$-axis.

Rotation: A rotation is a transformation in which an object is rotated around a fixed point called the center of rotation. The object can rotate the desired angle. The rotations can be made clockwise or anti-clockwise.

The angle $\alpha$ determines the rotations of the horizontal lines, while the angle $\beta$ arranges the rotation of the vertical lines. When $\alpha = \beta$ determines a rigid rotation on the origin and simultaneous reflection first through the $x$ axis and then through the $y$ axis is equivalent to making a single rotation of the object $180^\circ$ on the origin.

In three-dimensional space, equation 1 takes the form indicated in equation 4, and such that for all $k, c, b_1, b_3$ and $b_3 \in R$
\[
T : R^3 \rightarrow R^3 / \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & k \cos \alpha & -c \sin \beta \\ 0 & k \sin \alpha & c \cos \beta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}
\] (4)

where \(0 < \alpha < 2\pi, 0 < \beta < 2\pi\)

In the case of space, geometric objects can be generated by considering rotation around a given axis. This rotation affects only the points that belong to the plane perpendicular to the axis of rotation [9].

5.2 Classroom Practice: Fractals Objects Generation

The students begin with the design of a figure pattern that consists of an equilateral triangle where it is inscribed a circumference and inside of it another smaller equilateral triangle.

![Figure 1. Pattern.](image1)

Then, the parameters involved in equation 3 alternate in a random way and apply it to all the points of the pattern designed in Fig. 1, to obtain the graphs of related transformations in the plane [10].

![Figure 2. Friezes and mosaics.](image2)

In Fig. 2, the results of the successive applications of translation, reflection, reflection and rotation are observed.

In a second instance, students are presented with an activity in which they must choose a regular polygon and apply successive transformations on the plane. This determines an iterative process through which it asks them to obtain simple and partial friezes and mosaics.

![Figure 3. Hexagonal Pattern.](image3)

The students decide to work with a regular hexagon as a pattern and start with a very simple structure like the one seen in Fig. 3, and from it they obtain other objects of more complex shapes by successive iterations.

If the regular polygon of Fig. 3 is applied by the related transformations of equation 3, they can obtain friezes and mosaics as shown in Fig. 4.
Finally, students are asked to construct a hexagonal base prism as a pattern surface and generate a fractal geometry in three-dimensional space. Fig. 5 shows the pattern used to generate geometries of more complex shapes.

![Hexagonal base prism](image)

**Figure 5. Hexagonal base prism.**

Now, the students apply a decrease in the geometry of the prism of Fig. 5, for this they perform the subdivisions in the middle of a process of recurrence with the matrix of the transformation.

In Fig. 6 it is shown how the prism is gradually divided into several prisms, until a new structure is obtained [11].

Students perform the first three iterations by applying the affine transformations described in equation 4, and the results of the first sequence are observed in Fig. 6.

![First sequence with Hexagonal base prism](image)

**Figure 6. First sequence with Hexagonal base prism.**

The students continue making a series of iterations, and manage to build a three-dimensional figure that can be seen in Fig. 7 with views from space positions.
Finally, they manage to perfect the results observed in Fig. 7 and obtain the geometries of Fig. 8.

6 CONCLUSIONS

The use of technology made it possible to make more explicit the relationships established between linear applications, related transformations and fractal objects, since these entities develop in a universe that is very broad, rich and complex.

The process of construction of objects, their visualization, and their interpretation in mathematical terms has allowed us to relate aspects of particularization to reflexive abstraction derived from modes of representation in the construction of knowledge.

The iterative nature of fractals could be understood by applying basic definitions of mathematics, and taking into account the theoretical framework that was essential to design the conceptual model.

The integration of computational mathematics with the technological applications of the curriculum in engineering careers has led to a greater understanding of fractals, in the way that fractal mathematics can find order within patterns that seem complicated.

In this way, students can have different problematic situations and experiences that are complementary to the A & GA program to introduce them to permanent search for new knowledge; this process being the necessary threshold to initiate research activities and permanent updating.

The collaborative environment developed in the Basic Science Laboratory and the applied teaching methodology gave the students a series of strategies and skills to solve the activity presented, which resulted in an increase in interest and motivation.

After finishing the class, a qualitative survey to the students was carried out informally about the subject presented which shows that the pedagogic approach proved very interesting for them. In addition, the students acknowledged the importance and the advantage of using specific software to generate the fractals, and also valued the possibility of working in a multidisciplinary laboratory equipped properly.
REFERENCES


