MENTAL CALCULATION STRATEGIES USED BY PRE-SERVICE PRIMARY SCHOOL TEACHERS*

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Abstract

Mental computation is an important skill which is used even in the everyday life. It is a skill which can be developed by practice, to which primary school Mathematics has a decisive contribution. For correct and fast mental computations one needs to apply some learnt or invented strategies. Thus a primary school teacher should master many mental calculation strategies for all the four basic operations and she/he needs to be flexible in the use of these strategies.

In this paper we present a research made among 239 Primary and Preschool Pedagogy specialization students. They were asked to mentally calculate some operations, then write down the strategies they used. These strategies were analyzed and categorized. We counted the number of different strategies used by each student and classified these strategies in case of each operation. Also, some incorrect application of some strategies are presented. The results show that about one fourth of the respondents didn’t use any calculation strategies, they just mentally followed the written algorithms. At the same time, one third of the students used only 1 or 2 strategies. Regarding the strategies they used, we identified the most different strategies for subtraction, but the easiest operations for students were additions.

The results of this research highlight the necessity of teaching mental calculation strategies for future primary school teachers.

Keywords: mental computation, mental computation strategies, flexibility in mental computation, Mathematics Education, pre-service primary school teachers.

1 INTRODUCTION

Teaching numeracy in early primary grades is an important topic in mathematics. Nowadays, the efficient development of children’s number sense, beside the learning and remembering standard algorithms, encourages the use of mental computation strategies.

Doing mental computation actually means ‘working with the head’ [9], i.e. inventing and using strategies based on understanding the nature of the number systems and elementary arithmetic operations [18]. It consists of fact learning, mental calculations, and computational estimation, but they are not totally separable. In our study we mainly focus on mental calculations, which skill is not only an essential part of mathematics, but it also constitutes an important part of everyday life [13]. Mental computation is important for the development of the number sense and it also contributes to a creative and independent thinking [12].

Developing pupils’ number sense is one of the most important goals of teaching Mathematics. To achieving this goal, the teacher also needs to have good number sense [19].

In this paper we present a research about pre-service primary and preschool teachers’ mental computation skills. We studied the mental computation strategies used by the students.

2 THEORETICAL BACKGROUND

Mental computation has been defined as the ability to calculate exact numerical answers without the aid of calculating or recording devices [12], as well as the use of nonstandard algorithms for computing exact answers [17]. Mental strategies are different from the traditional standard numerical calculation algorithms, because they use numbers instead of digits, give opportunity for several mode
of doing calculations, and rarely use the paper and pencil techniques, since partial results are not written down [9].

Mental computation has been seen in two ways: as a basic skill or as a high-order thinking skill [13]. If we look at mental computation as a basic skill, it is just applying some learnt procedures when calculating without any aids. But seeing as a high-order thinking skill, students need to generate their own computation strategies [13].

There are studies highlighting the significance of mental computation in the first few years of school [3, 2], pointing out that it is strongly associated with number sense [11, 18]. The development of this skill contributes to the growth of cognitive and metacognitive thinking [10, 7], promoting the higher-level mathematical thinking [15].

In their study Gürbüz and Erdem [5] emphasize that the learner’s mental process is strongly related to mathematical reasoning ability. Its cultivation contributes to the development of students’ critical thinking [4], to a deeper understanding of number structure and properties; to the development of creative and independent thinking; to the development of the problem solving competence [14, 16].

The Romanian elementary school curriculum does not exactly highlight the development of mental computation, but emphasizes the deepening of quick calculations techniques, paying attention to the operations’ properties, or the way of splitting and grouping the numbers. We believe that it is important to focus on the systematic teaching of mental arithmetic strategies from an early age. Therefore, it should be implemented in the curriculum, without weakening the importance of standard algorithms’ teaching.

Mental computation strategies can be taught in two ways. One approach is by encouraging students to invent and share with others their own intuitive strategies for a given computation problem [8]. Based on the other approach there are few advanced strategies which should be taught to everyone in a special lesson [1].

3 METHODOLOGY

The research was conducted in the 2018/2019 academic year at Babes-Bolyai University, Romania.

3.1 Research goal

The aim of the research is to study pre-service primary school teachers’ mental calculations skills, focusing on the strategies they use.

3.2 Research questions

During this research we tried to find answers to the following questions:

- Q1. How many different strategies pre-service primary school teachers use for some given additions, subtractions and multiplications?
- Q2. Is the number of strategies used by a student correlated with the fact that he/she had Mathematics at A-level exam?
- Q3. Is the number of strategies used by a student correlated with the fact that he/she have teaching experience?
- Q4. What strategies pre-service primary school teachers use for calculating different operations (addition, subtraction, multiplication)?

3.3 Research participants

In this research 239 Primary and Preschool Pedagogy specialization students of Babes-Bolyai University have participated: 60 (25.1%) first year, 101 (42.3%) second year, 59 (24.7%) third year undergraduate students, and 19 (7.9%) first year masters students; 4 (1.7%) of them men. 12 (5%) of the respondents have teaching experience and 140 (58.6%) of the students had Mathematics exam at A-level exam. All the 239 students filled the questionnaire.
3.4 Data collection

In order to collect the data a questionnaire was used. The first part of the questionnaire contained 5 demographical questions (sex, age, study year, existence of Mathematics exam at A-level, existence of teaching experience), while the second part was consisted of 7 operations (see Table 1) to calculate mentally and explain what strategy they used. The aim of the questionnaire was to see if students use written calculation algorithms or mental computation strategies while calculating different operations mentally. The questionnaires were filled in on paper in the presence of the researchers.

<table>
<thead>
<tr>
<th>Table 1: Operations from the questionnaire.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Additions</strong></td>
</tr>
<tr>
<td>36+8</td>
</tr>
<tr>
<td>59+11</td>
</tr>
<tr>
<td>317-98</td>
</tr>
</tbody>
</table>

4 RESULTS

4.1 Mental calculation strategies used by pre-service primary school teachers

The second part of the questionnaire required performing some basic mental calculations and explaining the used strategies. The task consisted of two additions, three subtractions and two multiplications (see Table 1).

The obtained data were qualitatively analyzed. The founded strategies were identified, categorized and summed up along each operation, then relative frequencies were counted.

There are various mental math strategies, figuring by several different names in the research literature described in [6]. Here we will present a summary of the observed strategies used by students when attempting to solve the mentioned problems. We note that apart from the categorization outlined below, there exist another ways of categorizing the strategies.

4.1.1 Addition and subtraction strategies

Analyzing the obtained data, the following strategies for addition and subtraction were identified:

**Splitting** (partitioning, regrouping, 1010). When adding or subtracting two numbers, first split them by place values, then add/subtract the highest place values, and finally the next place value(s):

36+27=(30+6)+(20+7)=(30+20)+(6+7)=50+13=63, 76-24=(70+6)-(20+4)=(70-20)+(6-4)=50+2=52.

Observe, that this strategy is not convenient in subtraction when the ones of the subtrahend numbers are greater than the minuend (the case of transfer). This strategy is quite similar to the written algorithm.

**Stringing** (sequencing, jump, N10). Keep the first term and split the second term into units and tens, then add/subtract tens and units successively to/from the first term:

44+37=44+30+7 =74+7=81, 65-37=(65-30)-7=35-7=28.

**Bridging through ten** (A10). The second term of the operation is separated into two parts, so that one part with the first term gives a multiple of ten:

36+8=36+4+4=40+4=44, 74-7=(74-4)-3=70-3.

**Compensation** (N10C). One term is rounded to the closest multiple of ten and the result is compensated for the extra bit added or subtracted:

52+79=52+80-1=132-1=131, 317-98=317-100+2=217+2=219.

**Balancing**. Add or subtract a certain number to the first number of the operation to get the nearest tens, then add or subtract the same amount to the second number, too in such way, that the two interventions eliminate each other:


In Table 2 the frequencies and relative frequencies of using the above presented strategies by students are presented. In this table we calculated only the correct use of each strategies. There were students, who used some of the strategies incorrectly, we will give some examples in the discussion below.
Table 2. Frequencies and relative frequencies of the use of different strategies used for addition and subtraction.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Addition</th>
<th></th>
<th>Subtraction</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>36+8</td>
<td>59+11</td>
<td>Total</td>
<td>%</td>
</tr>
<tr>
<td>splitting</td>
<td>84</td>
<td>45</td>
<td>129</td>
<td>43</td>
</tr>
<tr>
<td>stringing</td>
<td>0</td>
<td>14</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>bridging through ten</td>
<td>69</td>
<td>85</td>
<td>154</td>
<td>51</td>
</tr>
<tr>
<td>compensation</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>balancing</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>157</td>
<td>144</td>
<td>301</td>
<td>100</td>
</tr>
</tbody>
</table>

The relative frequencies are also presented in Fig. 1.

In case of addition, as it can be seen in Table 2 and Fig. 1, the bridging through ten was the most preferred strategy (used in 51% of the cases), followed by splitting (43%). These two strategies were used in majority of the cases, representing 84% of the total of 301 instances of different strategies related to addition. The stringing and compensation strategies are observed only in a few cases (6% in total). Nobody used the balancing strategy.

As regarding subtraction, the most preferred strategy was the compensation strategy used in 46% of the total of 279 instances of different strategies. But some students didn’t use the method correctly, they rounded to tens the subtrahend (increasing with 1), then they subtracted 1 instead of adding it: 70 – 19 = (70 – 20) – 1 = 49 or 45 – 9 = (45– 10) – 1. In case of the operation 317-98 the use of this strategy gave the opportunity for more errors. Students rounded 98 to 100, but then subtracted 2 instead of adding it: 317-100-2. Also there were students who rounded 317 to 300, then subtracting 17 instead of adding it: 300-98-17.

Splitting accounted for 29% of strategy uses. In case of the operation 45-9 only one student used the splitting correctly, others made mistakes when changing the order of the terms and regrouping them: 45 – 9 = (40 – 9) – 5 = 31 – 5 = 26 or 45 – 9 = (45– 10) – 1 = 34. Splitting was the most used method for the operation 70-19, used by 77 students. There were even some interesting ideas, as 50+(20-19) or 60-10+10-9. The strategy bridging through ten was used for 20% of cases mostly for the operation 45-9. The frequency of use of stringing was only 2%, and for balancing 3%. In case of balancing the following mistake was observed: 317 – 98 = 325 – 90 = 225, i.e. the student, in order to subtract exact tens, decreased 98 with 8 and increased 317 with 8 instead of decreasing it. The same error was observed also for the operation 45-9, where students rounded 45 to 50, but they did mistake when balancing: 45-9=50-4=46 or 45-9=(45+5)+4=54. In these cases the students didn’t even observe that the difference is bigger than the minuend.

### Figure 1. Strategies used for addition and subtraction

#### 4.1.2 Multiplication strategies

In case of multiplication the following strategies were identified:
Doubling and halving. Double one factor and halve the other. It is useful when one of the factors ends with 5: 25×72=50×36=100×18=1800.

Splitting (partitioning, distributive property). This strategy refers to the idea that either one of the two factors in a product can be decomposed into two or more parts, and each part multiplied separately and then added: 3×67=3×(60+7)=3×60+3×7.

Compensation. Change one of the factors to an easy multiple of ten or hundred, carrying out the multiplication, and then adjust the answer to compensate for the change that was made: 7×19=7×20-7.

Using factorization. Factor the number before multiplying to make the operation simpler: 15×6=15×2×3.


In Table 3 the frequencies and relative frequencies of using the above presented strategies by students are presented.

Table 3. Frequencies and relative frequencies of the use of different strategies used for multiplication.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Multiplication</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15×6</td>
<td>416×25</td>
<td>Total</td>
<td>%</td>
</tr>
<tr>
<td>splitting</td>
<td>52</td>
<td>32</td>
<td>84</td>
<td>60</td>
</tr>
<tr>
<td>doubling and halving</td>
<td>0</td>
<td>9</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>using factorization</td>
<td>29</td>
<td>3</td>
<td>32</td>
<td>23</td>
</tr>
<tr>
<td>compensation</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>combination of methods</td>
<td>12</td>
<td>3</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>Total</td>
<td>93</td>
<td>47</td>
<td>140</td>
<td>100</td>
</tr>
</tbody>
</table>

The relative frequencies for different strategies are also presented in Fig. 2.

Regarding multiplication, a total of 140 instances of use of different strategies were identified (see Table 3). As it can be seen in Fig. 2, the most used strategy was the splitting (60% of the cases). It was the mostly used strategy for both of the multiplications. For 15×6, splitting 15 into tens and ones we get a similar algorithm to the written algorithm. In case of the operation 416×25, splitting was made in more ways: 416×10+416×10+416×5, 400×25+10×25+6×25, 416×20+416×5. The first splitting tried to use the advantage of multiplying with 10. The second and third splitting lead to the written algorithm. This could explain the popularity of this method. A student made an interesting error when used the splitting strategy: 416×25=(416×20)×5=832×5=4160, so when splitting 25, instead of using addition she used multiplication.

The second most used strategy was factorization (23% of the cases), mostly used for the operation 15×6, where factorizing as 15×2×3 makes the multiplication easier. In case of 416×25, the method doubling and halving was the second mostly used method, applied by 9 students. This is the most convenient strategy for this multiplication, as the operation 416×100:4 is easier to calculate.
4.2 Number of strategies used by students

Based on the above presented categorization of the mental calculation strategies, the number of different strategies used by each student was identified, then mean (M) and standard deviation (SD) was calculated. The mean is 1.91 (SD=1.695), which is very low, taking into consideration that these students are going to become primary school teachers. 65 students (27.2%) didn’t use any mental calculation strategy, 95 students (37.75%) used only 1 or 2 strategies, 59 students (24.69%) 3 or 4 strategies, and only 20 students (9.37%) used more than 4 strategies (see Table 4).

<table>
<thead>
<tr>
<th>Number of strategies used</th>
<th>Number of students</th>
<th>Percentage of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>65</td>
<td>27.2</td>
</tr>
<tr>
<td>1</td>
<td>46</td>
<td>19.25</td>
</tr>
<tr>
<td>2</td>
<td>49</td>
<td>20.5</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>13.39</td>
</tr>
<tr>
<td>4</td>
<td>27</td>
<td>11.3</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>5.86</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>1.67</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>0.84</td>
</tr>
</tbody>
</table>

To compare the number of strategies used by students with and without Mathematics at A-level exam, independent t-test was performed (Table 5). Even if the average in case of students with Mathematics exam is a bit higher (M=2.00, SD=1.811) than in the case of students without Mathematics exam (M=1.78, SD=1.516), the difference is not statistically significant: t(237)=1.970, p=0.319.

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>SD</th>
<th>N</th>
<th>p</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Mathematics</td>
<td>2.00</td>
<td>1.811</td>
<td>140</td>
<td>0.319</td>
<td>1.970</td>
</tr>
<tr>
<td>Without Mathematics</td>
<td>1.78</td>
<td>1.516</td>
<td>99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Independent t-test was used also for comparing the number of strategies used by students with and without teaching experience. The average of those students who has teaching experience (M=3.83, SD=2.290) is significantly higher, than of those without teaching experience (M=1.81, SD=1.607): t(237)=4.145, p=0.000 (Table 6).
Table 6. Comparing number of strategies used by students with and without teaching experience

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>SD</th>
<th>N</th>
<th>p</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>With teaching experience</td>
<td>3.83</td>
<td>2.290</td>
<td>12</td>
<td>0.000</td>
<td>4.145</td>
</tr>
<tr>
<td>Without teaching experience</td>
<td>1.81</td>
<td>1.607</td>
<td>225</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

In this research pre-service primary and preschool teachers’ mental calculation skills were studied focusing on the strategies they use. The results show, that most of the students don’t know many mental calculation strategies: more than one fourth of the respondents didn’t use any calculation strategies, they just mentally followed the written algorithms; and more than one third used only 1 or 2 strategies. Regarding the strategies they used, we identified the most different strategies for subtraction, but the easiest operations for students were additions. There is no significant difference between the numbers of strategies used by students with and without Mathematics at the A-level exam. The number of strategies used by those students with teaching experience is significantly higher than of those without teaching experience.

The results of this research highlight the necessity of teaching mental calculation strategies for future primary school teachers. Thus the research could evolve in the direction of identifying efficient teaching methods for developing mental calculation skills. Another direction to follow is to find out the reasons that pre-service primary schools teachers use a low number of mental calculation strategies by comparing samples from different countries having different approach for teaching mental calculations.

REFERENCES


