ROOT CAUSES OF THE MAIN DIFFICULTIES FOUND BY THE ENGINEERING DEGREE STUDENTS IN THE SUBJECT NUMERICAL METHODS

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Abstract

This research is part of the Teaching Innovation Project of the University of Oviedo "Reinforcement of basic mathematical concepts throughout engineering degrees. Detection of conceptual errors" (PINN-18-A-087)

Several authors note that certain errors are made by a high percentage of students in mathematics subjects, as has also been detected in this paper. This circumstance seems to be widespread not only in Spain, but also in several European countries. In the case of engineering students, several researchers have verified the existence of a deficient previous mathematical formation.

The reasons why students make mistakes in math can be very diverse. But when these errors are very similar between very different students, from different schools and without having had previous contact, the reason or reasons for these errors may be more limited. From this study we intend to study these errors.

In the studies carried out during the academic year 2018-19 by the Bacunimat group, formed by the teachers who applied for the Project referred to above, we highlight one of the conclusions obtained:

"The first-year students of the Engineering degree in the first four-month period come from high schools with some lack of conceptual knowledge of mathematical analysis. In the present Project it is verified that these insufficiencies improve after taking the subject of Calculus. In addition, no significant differences have been detected by the type of centre of origin, although they do depend on gender".

This paper analyzes the mathematical knowledge acquired by students of engineering degrees currently being studied at the Polytechnic School of Gijón (University of Oviedo), at the beginning of the second quarter of the first year. These students, therefore, have already taken the subjects of Calculus and Algebra and during the second term, they are taking Numerical Methods.

The research carried out is based on the analysis of the results of a questionnaire. The questionnaire consisted of a test on basic mathematical concepts, consisting of a total of 8 questions, which was passed on to the students in the first sessions of the Numerical Methods subject (January 2019). A total of 89 students responded to the questionnaire. The results obtained have made it possible to carry out an analysis of the degree of assimilation of the subjects of Calculus and Algebra by these students.

In addition, this paper also analyzes the relationship between the errors in fundamental concepts shown by students and the way in which these concepts are taught in high school courses.

Keywords: Virtual Campus, Mathematics, Numerical Methods, Calculus and Algebra.

1 INTRODUCTION

As Riviere said in a work published in 1990 [1], "mathematical learning does not consist of a process of incorporating data, rules, etc. into a blank mind, but implies a dialogue (implicit or explicit) between the student's previous knowledge and the new ones, which the teacher tries to teach him" Bearing in mind that in order to learn mathematics there must be a dialogue between prior knowledge and new knowledge, the hypothesis that poorly grounded knowledge can lead to "common" errors among students gains strength. What reasons can lead to poor prior knowledge?

One of the factors may be weak textbooks or conceptual errors. Analyzing textbooks from different publishers, we observe the lack of rigor and, in some occasion, errors in the conclusions reached, which can lead to conceptual errors in students [2].
From the historical point of view, in the transmission of knowledge, an important milestone has been the appearance of the textbook, which can be considered a cultural element that reflects the social manipulation that selects some contents over others, that imposes a certain way of structuring them and that proposes to the next generation certain types of problems with some semiotic tools and not others [3].

The systematization of methods that can lead to carelessness in understanding what is done, why it is done and for what. "The affirmation to the student that there is an automatic method to establish a family of results, even if it is true, tends to unload the fundamental responsibility of the control of intellectual work, blocks the transmission of the problem, which often causes the activity to fail" [4].

During the second year of high school in Spain, the pressure of the EBAU exam, the one that must be passed in order to have access to university, the breadth of the curriculum and the reduced teaching time, can lead teachers to systematize methods to streamline the acquisition of basic knowledge that allow students to pass the tests.

The learning of mathematical symbols and vocabulary is for many students a problem similar to the learning of a foreign language. A lack of semantic comprehension of mathematical texts is a source of errors. During the Secondary school, the use of mathematical symbology is not uniform between schools or between teachers. There are teachers who use it but do not demand it.

In the category of errors due to poor learning of facts and skills, all deficiencies in content and specific procedures for performing a mathematical task [5] are included.

The present work analyzes the mathematical knowledge acquired by the students of some of the engineering degrees (Electrical, Electronic, Mechanical, Industrial Chemistry and Industrial Technologies) that are currently being studied at the Polytechnic School of Engineering of Gijón (University of Oviedo), at the beginning of the second semester of the first year. These students have already studied the subjects of Calculus and Algebra and during the second semester, they will study Numerical Methods.

The research carried out is based on the analysis of the results of a test on basic mathematical concepts, formed by a total of 8 questions, which was passed to the students in the first class sessions of the subject of Numerical Methods. The responses of a total of 89 students to the questionnaire are available. The results obtained have allowed an analysis regarding the degree of assimilation of the subjects of Calculus and Algebra by these students, also taking into account if they have passed those subjects or not.

In addition, in the present work, the relationship between the errors in fundamental concepts shown by the students, with the way in which said concepts are taught in the bachelor-degree courses is also analyzed.

2 METHODOLOGY

The frequencies of the answers were analyzed through their number of occurrences and percentages. Mean values, medians and standard deviations were calculated. The normality of the data was studied through the Anderson-Darling test. Given the lack of normality of the distributions, for the non-parametric comparison among groups the Kruskal-Wallis test was used. Categorical variables were analysed with the help of Chi-squared test.

An 82% of the students under analysis were men and the other 18% women. A 55.1% of then came from public high schools, a 29.2% has already been enrolled at university the previous academic year and the other 15.7 came from either chartered or private schools. Figure 1 shows the test with the eight questions that were proposed to the students. Five of the eight questions proposed in this test (questions with numbers 1, 3, 4, 5 and 8), were already proposed by the authors to a group of students of the same degrees in the academic year 2017-2018 [6].
3 RESULTS

The subject of Numerical Methods is studied in the second semester of the first year of the EPI engineering degrees. In the first semester, these students have studied the subjects of Calculus and Algebra. Of the total number of students surveyed, 37 (41.57%) passed the Calculus subject, while 52 (58.43%) failed it. Regarding the subject of Algebra, 44 (49.44%) of the students passed it and 45 (50.56%) failed.

Table 1 shows the number and percentage of correct answers, failures and questions not answered in the test. The three questions that have a higher percentage of success among students are questions 1 and 2, followed closely by question 4. From the point of view of the authors, question 1 requires that students know the definition of logarithm and it is well-known from themselves. In the case of question 2, it seems to be a well-known concept for students. Question 4 addresses a concept that is already well known among high school students, but due to the way of explaining it, there is a tendency to confuse the value of the end of a function with the point of the domain where it is reached.

Figure 1. Test proposed to the students.

1) \( \log_2(1/4) = \)
   a) 2  b) -2  c) 1/2  d) does not exist

2) Function \( f(x) = 1/x \) defined in \( D = \{ x \in \mathbb{R} / x \neq 0 \} \)
   a) Does not strictly increase or decrease in \( D \)  b) Is not injective in \( D \)
   c) It is strictly decreasing in \( D \)  d) It is strictly increasing in \( D \)

3) The number of real roots of the equation \( x^2 + x - 5 = 0 \) is
   a) 0  b) 1 simple  c) 1 simple and 1 double  d) 3 simple

4) Let \( f(x) = x^3 \) defined in the closed interval \([-2, 1]\). The absolute maximum of \( f(x) \) is
   a) 4  b) -2  c) 1  d) there is no absolute maximum

5) If \( f \) is continuous in \([a, b]\), the area of the region under the curve \( y = f(x) \), and the vertical lines \( x = a \) and \( x = b \) and the abscissa axis, is given by:
   a) \( \int_a^b f(x)dx \)  b) \( \int f(x)dx \)
   c) \( \int_a^b f(x)dx \)  d) none of them

6) If \( f \) is continuous \([a, b]\) and \( \int_a^b f(x) dx > 0 \) then \( \forall x \in [a, b] \) it is verified that:
   a) \( f(x) \geq 0 \)  b) \( f(x) > 0 \)  c) \( f(x) = 0 \)  d) none of them

7) Let \( A, B, C \) three matrix such that \( AB = AC \). When can be say that \( B = C \)?
   a) always  b) if the three matrix are squared
   c) if \( A \) is squared and determinant different of zero  d) if \( B \) and \( C \) are squared and can be inverted

8) Let \( Ax = b \) a Cramer's equation system, in other words a linear equations system where \( A \) is squared and can be inverted. Which of the following methods is in general the most efficient for solving the system?
   a) Gauss  b) Gauss-Jordan  c) Cramer  d) Obtener \( A^{-1} \)

Please note that the most efficient is the one that requires of a less amount of elemental operations.

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Table 1. Number and percentage of answers: correct, wrong and not answered.

<table>
<thead>
<tr>
<th></th>
<th>Correct</th>
<th></th>
<th>Wrong</th>
<th></th>
<th>Notanswered</th>
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<tbody>
<tr>
<td>N</td>
<td>(%)</td>
<td>N</td>
<td>(%)</td>
<td>N</td>
<td>(%)</td>
</tr>
<tr>
<td>P1</td>
<td>42</td>
<td>46</td>
<td>51.69%</td>
<td>1</td>
<td>1.12%</td>
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<tr>
<td>P2</td>
<td>38</td>
<td>50</td>
<td>56.18%</td>
<td>1</td>
<td>1.12%</td>
</tr>
<tr>
<td>P3</td>
<td>12</td>
<td>71</td>
<td>79.78%</td>
<td>6</td>
<td>6.74%</td>
</tr>
<tr>
<td>P4</td>
<td>37</td>
<td>52</td>
<td>84.33%</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td>P5</td>
<td>30</td>
<td>59</td>
<td>66.29%</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td>P6</td>
<td>18</td>
<td>67</td>
<td>75.28%</td>
<td>4</td>
<td>4.49%</td>
</tr>
<tr>
<td>P7</td>
<td>27</td>
<td>30</td>
<td>69.94%</td>
<td>3</td>
<td>3.37%</td>
</tr>
<tr>
<td>P8</td>
<td>23</td>
<td>62</td>
<td>69.66%</td>
<td>4</td>
<td>4.49%</td>
</tr>
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On the other hand, the two questions that are failed for a largest number of students are questions 3 and 6. In the case of question 3, many students think that a polynomial of grade 3 should have 3 roots but they do not realize that the 3 roots are on the body of complex numbers and not of real numbers as it is asked in this question. About question 6, it is a question that requires from students a good knowledge about integration theory.

Table 2. Number and percentage of answers: correct, wrong and not answered (common question test of academic year 2017-2018) [6].

<table>
<thead>
<tr>
<th></th>
<th>Correct</th>
<th></th>
<th>Wrong</th>
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<tbody>
<tr>
<td>N</td>
<td>(%)</td>
<td>N</td>
<td>(%)</td>
<td>N</td>
<td>(%)</td>
</tr>
<tr>
<td>P1</td>
<td>39</td>
<td>33</td>
<td>41.25%</td>
<td>8</td>
<td>10.00%</td>
</tr>
<tr>
<td>P3</td>
<td>51</td>
<td>63</td>
<td>78.75%</td>
<td>5</td>
<td>6.25%</td>
</tr>
<tr>
<td>P4</td>
<td>25</td>
<td>49</td>
<td>61.25%</td>
<td>6</td>
<td>7.50%</td>
</tr>
<tr>
<td>P8</td>
<td>37</td>
<td>36</td>
<td>45.00%</td>
<td>7</td>
<td>8.75%</td>
</tr>
</tbody>
</table>

Table 2 shows the number and percentage of answers correct, wrong and not answered of those questions that were the same in the test proposed to the students of the academic year 2017-2018. Due to the low number of individuals that did not answered the proposed questions, they were not considered in the Chi-squared statistical test performed. The results obtained showed that the behaviour of students in questions 1, 3 and 5 was the same in the two academic years with non-statistically significant differences among years ($\chi^2 = 0.657, p = 0.418$; $\chi^2 = 0.073, p = 0.787$; $\chi^2 = 0.001, p = 0.992$) while there were statistically significant differences in the results obtained in question 4 ($\chi^2 = 8.863, p = 0.003$) and 8 ($\chi^2 = 9.307, p = 0.003$).

Table 3 shows the number and percentage of students that chosen each one of the answers. There are some incorrect options that were chosen more frequently that the right ones. It happens in the cases of question 2 with answer c, question 3 with answers a, c and d, question 5 answer b, question 6 answers b and c and question 8, answer b. This fact shows how certain mistakes are common to many students.

In question 2, a great number of students consider that function $f(x) = \frac{1}{x}$ is decreasing in its whole domain. Please note that the referred function decreases in intervals $(-\infty, 0)$ and $(0, \infty)$. Considering that this function decreases in $(-\infty, 0) \cup (0, \infty)$ is a mistake that can be solved if students realize that $f(-1) < f(1)$.

In question 3 only a 13.5% of the students selected the right answer. According to our experience, many students are confused about the difference between irrational and complex roots. From a theoretical point of view, and in order to provide a right answer to this question, it would be useful for students knowing both Role and Bolzano theorems, but from a practical point of view, it would be enough just studying the derivative of the function as any polynomial of odd degree has, at least, a real root. In spite of these and taking into account the experience share with ourselves of some secondary school teachers, many students consider this question just as an algebraic problem.
In question 5, relative to the calculation of the area of a flat region, a 40% of the students propose that the area coincides with the absolute value of the integral of the function. From our point of view, this mistake can be related to two fundamental reasons; on the one hand, the lack of mastery of mathematical language and on the other, the fact that the absolute value of a number is always an amount greater than or equal to zero. It is not difficult to graph functions that change sign in a range \([a, b]\) in such a way that the absolute value of the definite integral does not coincide with the area enclosed by the graph of the function and the abscissa axis.

In question 6, only a 20% of the students choosen the correct answer. One of the properties of the definite integral states that if the integrating function is positive in the interval \([a, b]\), then the corresponding integral defines a positive real number. The majority error of the students consists in thinking that the opposite implication is also verified.

In question 8, most of the students chooses the Gauss-Jordan method as the most efficient method to solve a Cramer system; however, the one that requires fewer operations is Gauss's. The students have worked on the resolution of systems using the Cramer's rule and the Gaussian method, although the latter is not usually explained in a systematic way in the baccalaureate. Textbooks also mention the so-called reverse matrix method, which we believe should be discarded from the beginning, and the Gauss-Jordan method.

The average number of hits per student was 2.55 with a standard deviation of 1.51 and a median value of 3. None of the students guessed the eight proposed questions, being the maximum number
of correct questions of 7. Figure 2 presents the distribution of the students in the sample according to the number of correct questions.

Anderson-Darling normality test was applied to the marks obtained by students. Due to the results obtained (AD=2.568, p<0.005), normality hypothesis was rejected, therefore, non-parametric tests were applied.

It was found that there were no significant statically differences in the total number of points obtained by students considering if they have either passed or failed the subject of Calculus (H=2.5, p=0.114). On the other hand, statistically significant differences were found in the median of the number of questions when those that passed and failed the subject of Algebra were compared (H=4.76, p=0.029). Those who passed Algebra obtained a median mark of 3 and those that not a median of 2. When groups compared, non-statistically significant differences were found neither by gender (H=0.45, p=0.503) nor by center where they come from (H=2.15, p=0.342)

4 CONCLUSIONS

The results obtained allow us to affirm that there are no significant differences in the level of knowledge shown by the test in the group of students analysed or in terms of their gender, center where they come from, or if they have passed the subject of Calculus or not. However, there are statistically significant differences depending on whether the students have passed the Algebra course. Note that the test was conducted during the first sessions of Numerical Methods class and, therefore, at that time the students still did not have a grade in the subject, so this test can serve as a way of early detection of those students who may experience difficulties in the subject.

REFERENCES


