T.O.MM. FOR ALL. TESSELLATION ORIGAMI MULTIMODULAR EXPERIENCE TO LEARN MATHS AND DEVELOP REPRESENTATION SKILLS

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Abstract

The use of origami technique for didactical aims is more and more investigated in these last years. Moreover, it is well known that a way to learn is to play. In this paper we propose an educational project involving mathematics and representations through the design of an origami puzzle game. Starting from the folding of a traditional origami model, we use its variations to design a tessellation that becomes a game and we also give a guideline to repeat the process starting from other models. We explain the methodology to choose the best starting model and to design how transform it in a multimodular experience in the learning process. Our principal aim is the education to the shape reading and to the development of a geometric language.

Keywords: Geometry, tessellation origami, symmetry, cognitive artefact.

1 INTRODUCTION

Our work starts from our common academic position within Turin Polytechnic, a context in which our approaches are often transversal to the sectors to which we belong, (see [1], [2], [3] and [4]). Remark that the use of origami technique for didactical aims is more and more investigated in these last years. Experimentations have been done in different school levels, from kindergarten to University, and about several disciplines, from scientific to humanistic fields [5], [6]. Moreover, it is well known that a way to learn is to play [7]. In this paper we illustrate our idea to use origami as a tool to design a game in order to involve students in an activity which allows to work on geometry, both from the point of view of mathematics and of representation. The chosen media is origami, because it is able to surprise participants -children for its magic and older students for its effectiveness-: it is the ideal tool to show geometry in a seemingly informal but also extremely rigorous way. Moreover, the use of the colored paper allows students to fold a lot of funny pieces in a cheap way.

T.O.MM. for all is an idea of concrete geometry [8] where it is possible to design a tessellation origami multimodular experience while learning maths and developing representation. The proposed activity, depending on the language used and the level of detail chosen, is part of the teaching program of the last years of primary school and of the first year of lower secondary school. moreover, we introduce a series of considerations that open, in a vertical and horizontal way, possible elaborations especially in its design declination. The laboratorial dimension of the proposal makes it possible to plan a series of coordinated, not-binding but synergistic activities, which range between vision and representation, design and calculation.

From the didactical point of view, our aims in this activity are:

- learn geometric language through the modeling of paper module;
- educate to the shape reading by decomposing and recomposing it;
- educate to understand the relationship between objects, symbolic representation and descriptive representation;
- stimulate the approach to game design

In this paper, we describe our methodology to design the tessellation’s pieces, we suggest some activity teachers can do with students and we collect our conclusion about the experimentation of T.O.MM. in different context.
2 METHODOLOGY

We describe the methodology of our work in order to design T.O.MM, as a tool that allows students to develop geometric skills. The choice of the model, the critical investigation about all its variations and the selection of the ‘frame’ useful to construct in the T.O.MM experience are proposed as guideline to repeat the process restarting from other models.

Below, our design path, obviously influenced by the aims with which we made our choices: our main focus was symmetries, between mathematics and representation. The main focus of the project is to explain the development of the activity. The first step to design this activity are the choice of the starting model, looking out to find the best diagram for its use, and the design of the folding sequence, in relationship with the lesson focus and the specific language to explain it.

2.1 The choice of the model

We chose the Windmill model because it is an extremely widespread model, normally used in its playful dimension: it is in fact one of the possible positions of the traditional model known as multiform, with very few steps it takes different forms: from windmill to table, from fish to various types of boats. The folding sequence is very easy and suitable for possible variations that privilege geometric aspects which are fundamental in this application proposal. The finished model has a rotational symmetry and during the bending sequence it acquires several conformations that allow many geometric observations. Folded starting from a square sheet, it is easy to manage even at very small dimensions, thus offering multiple levels of detail.

Last but not least, the individual model is already a complete object in its own right and therefore learning its folding sequence directly offers a play that can be used independently, thus favouring the repetitiveness of its folding and consequently the exercise.

2.1.1 The choice of the folding sequence

For a given origami model, it is possible to find different folding sequences, depending on the style of origami artists. For example, Fig. 1 shows two different ways to fold the windmill. From the point of view of our methodology, the choice of the folding sequence is very important both for the mathematical topics we want to visualize during the folding process, and for the folds appearing in the final model.

We exemplify this concepts discussing the two different folding sequences proposed in Fig. 1. For example, from a mathematical point of view, diagram (a) can be easily used to discuss polygons and their properties (in particular for shapes appearing in passages from 8 on) and diagram (b) to discuss central and axial symmetry. Relative to the final model, both models introduce the two diagonals of the initial square that appear in the back side of the final model that we don’t need. As we will show in section 3.1, we decide our folding sequence to underline the symmetries, as mathematical topic, and to avoid extra folds in the windmil.

![Figure 1. Different folding diagrams for windmill base.](image)

From left: (a) [www.origami-resource-center.com](http://www.origami-resource-center.com); (b) diagram by Frantisek Grebeníček.
2.2 The critical investigation about all its possible variations

The multiform/windmill, at its last step, offers a model with a rotational symmetry with those that, from now on, we will call “winglets”.

Having said that the winglets are born as an overturning of the excess paper, after the series of folds that brings the basic square with \( l \) edge and area \( l^2 \) to become a square with a \( l/2 \) edge and area \( l^2/4 \), their position can be twofold, in what we will call left (L) or right (R) position (as in Fig. 2).

![Figure 2. Basic module, 4 winglets. From left, module with winglets all in position L (left), module with winglets all in position R (right).](image)

It is the management of the winglets that characterizes the instruments of the game since they not only determine the shape but also allow to connect pieces: with their triangular shape, with sides equal to \( l/4, l/4 \) and the \( l\sqrt{2}/4 \), to game purposes, they can be inserted in the 'pockets', defined by the different folded layers of paper, making different pieces integral with each other.

2.2.1 From the traditional model to its possible variations, the square module

The position of the winglets (L/R) allows the basic model to assume different conformations coming to define a series of variations to the basic module without altering its geometric characteristics: each piece remains square, with four projecting winglets (Fig. 3).

![Figure 3. Basic module, 4 winglets. From top left, module with winglets all in position L (left), to bottom right, module with winglets all in position R (right).](image)

Actually, the variations can be many more because the winglets do not have to be protruding and can be folded and hidden under the central body. E.G., in Fig. 4, are summarized all the possible conformations of the variants of the first of the 16 models shown in Fig. 3, the module with 4 left winglets.
To complete the picture of the possible variants it would be necessary to make the same observations for each of the other models shown in Fig. 3.

Figure 4. Basic module, variants of tesserae with 0-4 winglets. The particular alignment and colour of tesserae in the drawings shows in the figure the permanence of certain winglets in the different configurations.

2.2.2 From the traditional model to its possible variations, other polygons

The critical analysis continues by evaluating the hypothesis of hiding not only the winglets but also part of the square surface of the outcome of the folding process, obtaining 3-4-5-6-sided polygons, with winglets and pockets useful for the management of the joints between the pieces (Fig. 5).

Figure 5. Processing of the basic Module, variants of tesserae with n winglets:
(a) digital representation of the tesserae polygonal central body: triangular, rectangular, pentagonal (in two different configurations) and hexagonal; (b) origami tesserae
2.3 The selection of the ‘frame’

For the purposes of the game, which and how many forms can be created by jointing (and not only by juxtaposing) the modules created? How are the shapes to be recreated as a puzzle proposed? Also in this case it is necessary to think about the purpose of the activities and the target of the users.

2.3.1 How many and which images?

Our purposes, where the focus is on symmetries, have led us to choose a series of figures that range from the presence of a single axis of symmetry to the presence of multiple ones and, at last, to present figures completely without symmetry (Fig. 6). The chosen figures are the result of a process of observation and recognition of simple forms as a simplification of complex shapes in a path that accompanies users to the selection of synthetic geometries.

2.3.2 Graphic language

Considering that the proposed figures are all traceable to real forms, the game could also start from the presentation of real physical objects or from photographs of the same but the resulting complexity of the game would be very high and the results would not be comparable because they are strictly bound to the individual abilities of abstraction. The objects to be reconstructed as puzzles are then proposed in the form of images and the graphic language used is part of the training experience itself.

Four levels of representation with different levels of complexity have been identified:

- colored tesserae (for immediate recognition of the polygon) and joints;
- colored tesserae (for immediate recognition of the polygon);
- tesserae
- outline

However, it should be remembered that each module is ideally a shape already but not necessarily all users see the same one (Fig. 5): in the design phase we found ourselves defining the individual modules as heart, diamond, cat's head ... and in the end the cat's head has really given rise to the cat model demonstrating how the ways to propose a model are endless. Naming a form is not an objective tool for users to recognize it and it is therefore necessary to proceed with its representation to make the object/puzzle association unique.
3 ACTIVITIES

Starting from the folding of the traditional windmill model, our steps, declined according to the age and the knowledge of students, are:

- description of geometry elements during the folding process taking in account topics of the students’ curricula (from the basic fact of recognize shapes to the analysis of their geometrical properties);
- description of complex shapes during a tangram-based game: we ask students to read shapes that we propose or they create, with different levels of complexity, from shape to outline and vice-versa;
- design of game, defining rules and contents, involving also other disciplines; the activity can be managed alone or within the institute assigning responsibilities to the different age groups that work synergistically for the common result, creating all the parts of a new game.

3.1 Mathematical lesson

We suggest a maths activity during the folding. For other examples of lesson see [1].

In this lesson, we focalize our attention on symmetry axis. We give the folding instructions for the windmill base, proposing a mathematical analysis on symmetries.

The starting sheet is a square (we suggest to use edges from 12 to 15 cm); it has four symmetry axes: the two diagonals and the two medians. Teacher can show the axes folding them on a square.

Now, follow the suggested folding steps.

1 Fold one of the symmetry axis parallel to a square edge and unfold, Fig 8(a).

2 The square is now divided in two rectangles. Each of them has only two symmetry axes: the two medians. Choose one of the two rectangles and fold the longest symmetry axes, Fig. 8(b). We get a bicolored rectangle showed in Fig. 8(c).

Figure 8. Folding instruction for windmill base.

2 Fold one of the symmetry axis parallel to a square edge and unfold, Fig 8(a).

2 The square is now divided in two rectangles. Each of them has only two symmetry axes: the two medians. Choose one of the two rectangles and fold the longest symmetry axes, Fig. 8(b). We get a bicolored rectangle showed in Fig. 8(c).
3 Fold in such a way that the first fold became a symmetry axis in Fig 8(c). We get a coloured rectangle in Fig. 8(d).

4 Fold the shortest symmetry axes of the coloured rectangle (Fig 8(d)) and unfold. The figure is divided in two squares, Fig 8(e).

5 Fold one of the squares along the symmetry axes parallel to the segment folded in step 4 obtaining Fig 8(f). Ask student to fold in such a way that the segment folded in step 4 became a symmetry axis, the fold is suggested in Fig 8(f).

6 Unfold the last two folds obtaining Fig. 8(g). To complete the base, we have to create mountain and valley folds following the instruction in Fig 8 (g) obtaining the final Fig. 8(h).

This is a very interesting point to discuss symmetries. We suggest fold only one corner at the beginning and then ask students to fold another one as they want in order to obtain a symmetric figure.

![Figure 9. Symmetry cases in the last passages.](image)

Fig. 9(a) shows what we obtain folding one corner. Students can propose new folds in order to obtain symmetrical figures; we describe symmetries in each case:

- Fig. 9(b): axial symmetry; the figure is symmetric with respect to a vertical axis.
- Fig. 9(c): axial symmetry; the figure is symmetric with respect to a horizontal axis.
- Fig. 9(d): central symmetry; the figure is symmetric with respect to an internal point.

In each case, students can verify the symmetry folding the paper; be careful that, in the central symmetry case, they have to compose two axial symmetries, folding the model along the vertical and then the horizontal axis.

### 3.2 Drawing as a graphic trace

The activity offers the possibility of making concrete reasoning on the relationships between object and model, drawing and geometric figures. The modular elements, made during practice, become cognitive artifacts to prepare the mind to welcome intuitions and discoveries. The activity offers the possibility of proposing the design to get to understand the shape and then model it or start from the model to observe details that on the drawing were not so obvious.

This is the case, for example, of the tortoise and crab models (Fig. 10) which, if shown as a simple outline, do not show what emerges from the observation of the models: they are the same tesserae offset with respect to the symmetry axis.

![Figure 10. Turtle and Crab. From left: (a) digital representation, outline; (b) paper modelling.](image)
The use of a specific language to support direct observation, calibrated according to the participants, allows students to develop geometric skills.

### 3.3 Design of game

The activity described in section 3.2 can be organized as a game, also dividing students in teams. We want to suggest two more games; teachers can design other proposals depending on the school level and on the students’ interests.
3.3.1 T.O.MM I want You!

In this game students are divided in teams. Teacher shows an outline and they have to win the necessary cards to complete the figure by answering questions. Teacher can choose the topic of the questions among one or more subjects.

3.3.2 T.O.MM trivial game

Student design a board using colored tesserae. They move around the board by correctly answering trivia questions which are split into six categories, combined with the tesserae colors. In the classical version of Trivial Pursuit we get these couple topic/color: History/yellow, Geography/blue, Science&Nature/green, Arts&Literature/purple, Entertainment/pink, Sports/orange.

4 CONCLUSIONS

The possibility of expressing the experience by a double point of view, math & Art, with the same language let bring us in an exploratory journey in concrete Geometry. The use, by the teaching staff, of a specific language calibrate according to the participants, to support direct observation, leads to the development of geometrical abilities that become the basis for subsequent implementations throughout the entire educational path, thus becoming part of a vertical distribution of knowledge.

The considerable flexibility of the proposed activity, suitable to be redesigned according to the possible users - classes, families, targeted homogeneous groups - allows its meaning "for all": the simplicity of the folding pattern of the basic model, the tangibility of the physical model, the direct relationship object / representation - and therefore its possible application even with visually impaired users - the role of memory - not only physical but above all experiential - make the TOMM experience a circular activity, never complete and always in progress in which it is possible to find ideas for subsequent application declinations.

The method is generalizable starting from other origami models in order to design new game involving students in activities to learn geometrical subjects.

We tested these activities with students of primary and middle school and in training days for teachers (see for example origamialelamio.altervista.org). Children and educator were very interested and involved in this activity. We test this game also in occasion of the big events “Il Salone del Libro di Torino”, for general public and we will test it in special workshop for schools.

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