EDUCATIONAL SOFTWARE TOOL FOR DIFFERENT WAVELET BASED METHODS: WAVDEC

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Abstract

In this paper Matlab-based application-GUI "Wavdec" is presented, serving as educational tool for different signal processing courses. Wavdec analyses an image into different wavelet subbands, using three different wavelet decomposition/reconstruction methods: Discrete Wavelet Transform (DWT), Quincunx transform and Steerable Pyramid Wavelet Transform (SPWT). Different levels and wavelet types for each method can be chosen. Also, Gaussian noise with manually defined mean and variance can be added to the original image. Hard thresholding, which either keeps or removes values of coefficients, for each subband can be defined arbitrarily. Thus, Wavdec can be used in different scenarios, for example to demonstrate scalable image compression or noise removal. Finally, Wavdec application automatically calculates image quality between original and reconstructed image using three image quality measures as well as their compression ratio. Finally, the effects of using the described original developed tool on education of electrical engineering students have been disclosed.

Keywords: educational tool, wavelet methods, Wavdec, signal processing, image quality.

1 INTRODUCTION

Fourier's theory is known to show the signal as a linear combination of sinusoids of various frequencies, as presented in [1]-[4]. However, signal analysis can be performed through other transformations, which, along with the frequency response of the signal, also show the time response [5]-[7]. Wavelets are a special kind of function used to analyse the signal, where analysis is performed by introducing signals in the form of a linear wavelet combination [8]-[10]. Such wavelet functions are associated with two independent variables (t - translation and s - scale) which determine timing of the appearance of individual spectrum parts.

The time-frequency representation described in this work divides the signal into several parts and then separate analysis is made for each of them. The problem of signal division is solved by using scalable windows that moves over the signal. The spectrum is calculated for each window position, and the procedure is repeated with a wider window for each period.

2 WAVELET BASED METHODS

For different applications, it is possible to use discrete wavelet transformation methods or continuous wavelet transformation [11]. When using the continuous transformation method, redundancy occurs due to the translation and scaling of the wavelet function over the entire signal. In addition, the infinite number of wavelet functions required for accurate transform transformation can be a problem, i.e., for most functions there is no exact analytical solution. Therefore, the discrete transformation method was used in this paper.

2.1 Discrete Wavelet Transform

Discrete transformation is obtained when discrete waveform signal transformation (DWT) is used, and the wavelet coefficients (wavelet decomposition) appear as a result. Discrete wavelet can be defined as a continuous function:

$$\psi_{q,k}(t) = \frac{1}{\sqrt{s_0^2}} \psi(\frac{t - k r_0 s_0^2}{s_0^2})$$

(1)

where q and k are discrete variables, constant $s_0$ is step of scaling, and $r_0$ is the translational factor.

By choosing $s_0 = 2$, sampling scheme on the axis of the scale is called the dyadic, and by selecting the $r_0 = 1$ timing sampling scheme also becomes dyadic.
The output signal can be reconstructed as a sum of wavelet functions multiplied by wavelet coefficients and thus defining inverse wavelet transformation as:

\[ f(t) = \sum_k \sum_i a_{q,k} \psi_{q,k}(t) \]  

(2)

where \( a_{q,k} \) are the wavelet coefficients, and \( \psi_{q,k} \) are the wavelet functions.

If the orthogonality of the wavelet function is met, redundancy occurring with continuous wavelet transformation is removed. Still, the problem of the infinite number of wavelets remains, whose resolution requires the application of the Fourier transformation:

\[ F\{f(at)\} = \frac{1}{|a|} \psi\left(\frac{\omega}{a}\right) \]

(3)

It is obvious that shortening the wavelets by factor \( a \) in the time domain expands its frequency spectrum for the same factor. In the case of scaling, there is a shortening and stretching by factor 2 with shifting the center of the spectrum for the same factor. Therefore, the spectrum can be covered by scaled wavelet, and with proper wavelet design, the spectrum should touch and overlap as shown in Fig. 1.

![Figure 1. Impact of scaling in the time domain on spectrum in the frequency domain.](image)

Since a wavelet is considered as a bandpass filter, starting from a wavelet and stretching it for factor 2, only the half of the low frequency band is covered. Consequently, wavelet needs to be extended to infinity to cover the entire spectrum until \( f = 0 \). The solution of the problem is the use of a scaling function that has a low frequency characteristic. The analysis is then performed from a maximum to a certain scale height, and the rest of the spectrum is covered by a scaling function as shown on Fig. 2.

![Figure 2. The use of the scaling (\( \varphi \)) and wavelet (\( \psi \)) function.](image)

It is shown that the wavelet transformation can be observed as filtering the signal with a set of filters, and the procedure itself is known as a subband coding. The scaling function then represents the impulse response of the lowpass filter, and the wavelet function is the impulse response of the highpass filter.

Discrete wavelet transformation is defined by connecting wavelet (\( \psi \)) and scaling (\( \varphi \)) functions on different scales. Thereby, the scaling and wavelet function of multiple scales can be expanded in terms of the basis scaling functions of the next higher resolution, and such functions are described in [12] (Fast Wavelet Transform). If the scaling and wavelet functions are orthonormal, the approximation and detail coefficients of the next scale (j-1) can be obtained by convolving and downsampling by 2 with approximation coefficients (or input signal in the beginning) of the previous scale j (i.e. by filtering with FIR lowpass and highpass filters):

\[ c_{j-1}(k) = \sum_n h_{\varphi}(n - 2k)c_j(n), \quad d_{j-1}(k) = \sum_n h_{\psi}(n - 2k)c_j(n) \]

(4)

For the discrete signal \( c_j(m) \) at the input of the filter, \( c_{j-1}(m) \) is the output signal of lowpass filtering, called the approximation (scaling coefficients) of signal \( c_j(m) \), and \( d_{j-1}(m) \) is the output signal of high frequency filtering signal i.e. the detail (wavelet coefficients) of signal \( c_j(m) \). The decomposition can
also be performed at more than one level, so the transformation is applied to the approximation coefficients. Further decomposition corresponds to the further division of the spectrum into a lower (approximation) and higher (more detailed) part. This process is often called a dyadic wavelet subdivision (other decomposition schemes also exist).

The approximation coefficients $c_j(m)$, represented by scaling and wavelet functions of the higher scale $(j-1)$, can be then reconstructed using inverse wavelet transform. Procedure is similar, but uses QMF (Quadrature Mirror Filters from $h_0$ and $h_1$) filters and sum of the convolutions with upsampled (by 2) approximation and detail coefficients from the higher scale.

Fig. 3 shows decomposition and reconstruction process for 2D input (i.e. for image). As DWT is orthogonal transform, separate one-dimensional decomposition and reconstruction can be done along rows and then along columns (or vice versa).

![Figure 3. Decomposition and reconstruction in two directions (horizontal and vertical), 1 level.](image)

### 2.2 Steerable Pyramid Wavelet Transform

Steerable Pyramid Wavelet Transform (SPWT) [13] is useful for image decomposition and for some applications gives better properties than the orthogonal wavelet transformation described earlier. Bands obtained by SPWT are independent of translation and rotation, unlike the wavelet transformation described earlier. Also, SPWT is reversible in frequency domain, i.e. it is possible to reconstruct the input signal. The decomposition and reconstruction scheme with SPWT is shown in Fig. 4.

In the first step, the image is divided into a low pass and high pass through $L_0$ and $H_0$. Thereafter, the low pass section is divided into oriented subbands through the $B_{0,k}$ filters, (over which the cross-correlation of one dimension is carried out in the horizontal and vertical direction) and the lowpass subband (obtained with the factor two in the horizontal and vertical direction). The recursive division for further subbands is obtained by inserting the shaded part into the place marked with $\bullet$ in Fig. 4.

The drawback of such a transformation is a problem of filter design, which cannot be implemented in the spatial domain, so a perfect reconstruction is possible only in the frequency domain. However, practical realizations (with only a minor additional error in reconstruction process) exist in spatial domain, for 1, 2, 4 and 6 directions and 2D input [14].

In addition, the transformation yields more output coefficients, than the original ones.
3 RESULTS

The software tool that uses different wavelet transformation is created in MATLAB [15] during this research. All the students attending the Digital Signal Processing course have been trained to use this software, i.e. to find the parameters that will give the best results. Standard test images "Lena" and "Baboon" were used [16]. For DWT, used toolbox in Wavdec can be downloaded from [17], for SPWT, used toolbox can be downloaded from [14] and for Quincunx transform from [18].

3.1 The application of DWT transformation

Two-dimensional DWT is calculated using two one-dimensional transformations. The application of such a transformation to the matrix representing the image, divides the image into four subbands. The first one-dimensional DWT divides the spectrum into two parts and the other DWT into two more parts, with another spatial orientations (Fig. 5). The result can be presented as four matrices:

- LL - approximation of the original image in the horizontal and vertical direction,
- HL - an image of horizontal details of vertical approximations,
- LH - an image of vertical details of a horizontal approximation,
- HH - an image of the details in horizontal and vertical direction.

![Figure 5. Two-dimensional DWT applied on the image.](image)

The decomposition can be further resumed with each band separately, and if N is the number of decomposition, the result is $4^N$ subbands. Continuing the decomposition only with approximation (LL matrix) is in fact, a dyadic decomposition and the result are $3N + 1$ subbands. Fig. 6 shows image...
reconstruction using only approximation subband images of different levels. This process is similar to scalable image compression, where each additional level adds new details in the reconstructed image (up to the original image).

![Figure 6. Reconstruction when using only the approximation coefficients: a) original image, b) 1-level decomposition (compression ratio 1:4), c) 2-level decomposition (compression ratio 1:16), d) 3-level decomposition (compression ratio 1:64)](image)

### 3.2 The application of SPWT transformation

An example of the 2-level and six directions image decomposition of Lena image is given in Fig. 7. Only the bandpass filtered images are shown (without highpass filtered image), and in the middle is the lowest band (using only lowpass filter).

![Figure 7. SPWT transformation of Lena image, 2 levels and 6 orientations.](image)
3.3 The application of Wavdec software tool

After running the Wavdec in Matlab environment, the application window opens (Fig. 8). Parameters that can be arbitrarily chosen are:

- approximation coefficients scaling (A1, A2, A3 depending on decomposition level);
- scaling the coefficients of the details (LH1, LH2, LH3 for details in vertical direction and the approximation in horizontal direction);
- HL1, HL2 and HL3 scaling are considered equal to the corresponding scaling for LH1, LH2 and LH3 subbands;
- HH1, HH2 and HH3 scaling for the coefficients of the details in both horizontal and vertical direction.

When reconstructing subband images into the original image, inverse operations are used, with hard thresholding to reject each subband coefficient. This means that all transformation coefficients that are smaller than the fixed threshold become 0. Furthermore, the level of decomposition can be chosen (level 1, level 2 or level 3), and the type of DWT wavelet filter (CDF9_7, Coif22_14 and Haar4). The Quincunx [19] and SPWT [13] wavelet transformations can be selected too. Quincunx transform can be used with different filters and at most 3 levels, while SPWT transform can be used with 1, 2, 4 and 6 direction spatial filters and at most 3 levels. By mouse clicking on the image of the reconstruction, the image can be sharpened, using a filter with different $\alpha$ ($0 \leq \alpha \leq 1$). Compression Ratio (CR) is here defined as a ratio of the coefficients that are different from 0, before compression and after decompression.

Image quality can be measured by various objective measures, from which Peak-Signal-to-Noise Ratio in dB (PSNR) and Structural Similarity Index Measure (SSIM) are used here [20]. Higher PSNR means that the examined image is more similar to the original image. SSIM can get values 0-1 where higher value means that the examined image is more similar to the original image (SSIM = 1 when tested images are the same) [21].

![Figure 8. Wavdec program, DWT transformation (decomposition and reconstruction) of Lena image, 1 level, perfect reconstruction.](image-url)
The following tables show CR and SSIM of the tested images (Tab. 1 for Lena image, Tab. 2 for Baboon image) with specified thresholds and decomposition levels. Program parameters were set as:

- A1, A2, A3 = 1
- HH1, HH2, HH3 = 1
- LH1, LH2, LH3 = 1
- Wavelet filter = CDF9_7, DWT transformation

### Table 1. CR and SSIM values for the image “Lena”.

<table>
<thead>
<tr>
<th></th>
<th>Threshold = 1</th>
<th>Threshold = 10</th>
<th>Threshold = 100</th>
<th>Threshold = 300</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR, 1 level</td>
<td>1:1.4656</td>
<td>1:3.2699</td>
<td>1:4.7812</td>
<td>1:24.2726</td>
</tr>
<tr>
<td>CR, 2 levels</td>
<td>1:1.5453</td>
<td>1:5.9079</td>
<td>1:15.615</td>
<td>1:21.8381</td>
</tr>
<tr>
<td>CR, 3 levels</td>
<td>1:1.5529</td>
<td>1:6.7654</td>
<td>1:47.8714</td>
<td>1:65.4052</td>
</tr>
<tr>
<td>SSIM, 1 level</td>
<td>0.99914</td>
<td>0.97877</td>
<td>0.81798</td>
<td>0.12522</td>
</tr>
<tr>
<td>SSIM, 2 levels</td>
<td>0.99904</td>
<td>0.96717</td>
<td>0.82135</td>
<td>0.6067</td>
</tr>
<tr>
<td>SSIM, 3 levels</td>
<td>0.99903</td>
<td>0.96452</td>
<td>0.73963</td>
<td>0.62781</td>
</tr>
</tbody>
</table>

### Table 2. CR and SSIM values for the image “Baboon”.

<table>
<thead>
<tr>
<th></th>
<th>Threshold = 1</th>
<th>Threshold = 10</th>
<th>Threshold = 100</th>
<th>Threshold = 300</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR, 1 level</td>
<td>1:1.1054</td>
<td>1:2.0331</td>
<td>1:4.2091</td>
<td>1:21.4802</td>
</tr>
<tr>
<td>CR, 2 levels</td>
<td>1:1.1176</td>
<td>1:2.415</td>
<td>1:15.3121</td>
<td>1:18.7406</td>
</tr>
<tr>
<td>CR, 3 levels</td>
<td>1:1.1191</td>
<td>1:2.4863</td>
<td>1:46.912</td>
<td>1:64.0626</td>
</tr>
<tr>
<td>SSIM, 1 level</td>
<td>0.99982</td>
<td>0.97329</td>
<td>0.75772</td>
<td>0.12438</td>
</tr>
<tr>
<td>SSIM, 2 levels</td>
<td>0.99981</td>
<td>0.96601</td>
<td>0.54191</td>
<td>0.44886</td>
</tr>
<tr>
<td>SSIM, 3 levels</td>
<td>0.99981</td>
<td>0.96487</td>
<td>0.43718</td>
<td>0.33587</td>
</tr>
</tbody>
</table>

The SSIM values of the quality assessment of the corresponding images (Lena and Baboon) for different types of wavelet filters are shown in the Tab. 3. Threshold was always set so that CR is 1:20 (± 0.05). Three levels were used, and all compression coefficients were set to 1 (A, HH and LH). It can be concluded that different wavelet filter gives somewhat different final image quality.

### Table 3. SSIM value for different filter types

<table>
<thead>
<tr>
<th>Filter</th>
<th>SSIM - Baboon</th>
<th>SSIM - Lena</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF9_7</td>
<td>0.62967</td>
<td>0.88126</td>
</tr>
<tr>
<td>Coif22_14</td>
<td>0.63397</td>
<td>0.8749</td>
</tr>
<tr>
<td>Haar4</td>
<td>0.61458</td>
<td>0.87057</td>
</tr>
</tbody>
</table>

Image denoising is shown in Fig. 9. By changing thresholds for different wavelet subbands, it is possible to remove noise in higher frequency subbands, while preserving important information from the original image in lower frequency subbands. Also, SPWT in this case gives better results than DWT. PSNR and SSIM values are given in Tab. 4.
4 CONCLUSIONS

The Wavdec software tool was created during the research work related to digital signal processing and image quality testing, and then applied for education purposes. During laboratory exercises with students of electrical engineering, it was noted that they successfully mastered the work with this software. By assigning different parameters in Wavdec program (Fig. 8), students could notice effects of applying different types of transformations and evaluate results, i.e. the quality of the processed image.

Concluding, it is possible and desirable to include undergraduate and graduate students in research conducted by teachers, i.e. that Education and Scientific Work are not two separate things, but they intertwine.

For research and education purposes, Wavdec application can be downloaded from [22].

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REFERENCES


