TAXONOMIES AND TECHNOLOGIES IN TRAINING  
(OF MATHEMATICS)  
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Abstract  
The objectives of the training are a leading component in the methodical training system, and in particular the Mathematics training and the main link between this system and the external environment. Therefore, any change in the external environment will affect the learning objectives, and hence the content, methods, means, forms of training and evaluation of achievements in Mathematics. The objectives of Mathematics training are justified by the general objectives of education, the very concepts of Mathematics, the values of mathematical education and their relevance to the social values, interests and needs. The present study is based on the concepts of Benjamin Bloom, George Poya and D'Eno about the taxonomy of learning objectives.  
The question is whether the external environmental changes at this stage are such as to influence the learning objectives and, consequently, the concepts of Bloom, Poya and D'Eno. The main change in the environment is that we live in an Internet society, a digital environment and electronic communication. We may also pose the questions, “Is there a need to change the already familiar teaching methods? Are these methods easy to change? Will the change lead to improved learning and more effective achievement of the goals?” Specific learning practices have been analysed on the basis of which hypotheses on methods and means to achieve higher levels of used taxonomies are put forward.  
Keywords: training, taxonomies, methodologies and methods of learning, effectiveness of training.  

1 INTRODUCTION  
It is generally recognized that for the training with a leading component in the methodical system of training and mathematical preparation, this system and external environment must be used. It closes every alteration of the external environment for the purposes of learning and also through them and the content, methods, tools, training and evaluation of mathematical achievements.  
The aim of Mathematics education is to take into account the objectives of education, the concepts of Mathematics, Mathematics and education, and their relevance to the public values, interests and needs. Weighing on the completeness of the presentation of the place and role of Mathematics, and in particular of Mathematics training, the practical impossibility of performing tasks related to the handling, the unknown variety and the unlimited application of Mathematics and Mathematical knowledge. There are a number of systematizations, classifications or simply extracting information and training content, as well as various practical education systems that can be considered as a reflection of risk and of Mathematics education.  
Standing on this ground and according to the assessment methodology, all the studies that are recognized as authors – Benjamin Bloom and George Pólya and D'Eno – will be presented.  

2 METHODOLOGY  
In 1956, Benjamin Bloom, a psychologist at the University of Chicago, published a taxonomy of educational goals [1], which over the past decades has been unusually valuable for the characteristics and results of teaching. Pedagogical taxonomy is the theory of creating a lean system of pedagogical goals, broken down by categories and hierarchical levels. Bloom's taxonomy has been developed to teach teachers how to classify a given learning assignment and how to define and grade learning objectives. For example, remembering certain information - concept, statement, principle, algorithm, main task, etc., however important they are, is at a lower level than the ability to analyze and evaluate. The symbolic taxonomy of Bloom is expressed by the pyramid shown below.
While Benjamin Bloom's theory is universal and applicable in all subject areas, the theory of George Pólya refers to Mathematics training and its essential component, namely solving a mathematical task, but not only. George Pólya is a Hungarian mathematician who worked for much of his life in Switzerland and the United States. As a result of his many years of research on a wide range of mathematical topics, he developed generalized methods to solve problematic situations [2]. Pólya formulates four phases of the problem solving process:

1. Understanding the problem
2. Devising a plan
3. Carrying out the plan
4. Looking back

The taxonomy of D'Eno is characterized by the so-called "operational approach". The training is set forth by precise, clear and detailed criteria and objectives for each topic. Taxonomy examines four groups that are leading the training:

1. Students activities – remembering and reproducing information, using algorithms, applying rules and solving problems;
2. Educational content – structure and interrelations of the information that is transmitted to the students;
3. Degree of integration – the possibilities for practical application of acquired knowledge and skills as well as the speed and effectiveness of their implementation;

It is not difficult to observe the parallel and synchronicity between Bloom's taxonomy, Pólya phases and D'Eno groups, which is probably due to the feasibility and effectiveness of these theories.

The question we are asking is whether the external environment changes at this stage are such as to influence the learning objectives and, consequently, the concepts of Bloom, Pólya and D'Eno. The main change in the environment is that we live in an internet society, a digital environment and electronic communication. We may also ask questions, "Is there a need to change the already familiar teaching methods? Are these methods easy to change? Will the change lead to improved learning and more effective attainment?"

Acting as a modern Mathematics teacher. On the one hand (according to Pólya's concept), if one wants to develop mentality in their students, they must inspire an interest in the tasks and give the students a lot of opportunities for imitation and practice. On the other hand (according to the Bloom Concept), it is necessary to classify a given learning task and to define and grasp the learning objectives that are achievable through it. Let's take an example and look at a specific task from ed. Anubis, 12th grade, Preamble section [4]. The choice is not accidental - publishers and authors are well established, the textbook is used in a wide range of schools. The analysis of a task from this textbook may be called a conditional analysis of a representative sample.
**Task 1:** Find the values of the parameter $a$, for which the $x|x + 2a| + 1 - a = 0$ equation has a single solution.

The motivation of the students is to negotiate, apply and strengthen their knowledge and skills at the end of their high school Maths education. Learning goals should be as high as possible, as both the task and the preparation of students allow. Let’s introduce the solution itself:

**Solution:** Due to the presence of a module, we consider the following two cases:

1) **Case** $x \geq -2a$. The equation receive the form $x^2 + 2ax + 1 - a = 0$. The discriminative of this equation $D = a^2 + a - 1$ is non-negative in the interval $a \in (-\infty; -\frac{1-\sqrt{5}}{2}] \cup \left[ -\frac{1+\sqrt{5}}{2}; +\infty \right)$. For these parameter values the roots of the equation are $x_1 = -a - \sqrt{a^2 + a - 1}$ and $x_2 = -a + \sqrt{a^2 + a - 1}$. In order to be the solution of the source equation, the conditions $x_1 \geq -2a$ and $x_2 \geq -2a$ must be fulfilled, namely $-a - \sqrt{a^2 + a - 1} \geq -2a$ and $-a + \sqrt{a^2 + a - 1} \geq -2a$. Solving the resulting irrational inequalities (each of them is reduced to a system of two rational inequalities), we observe that $x_1$ is the solution of the equation for $a \in \left[ -\frac{1+\sqrt{5}}{2}; 1 \right]$, and $x_2$ is the solution for $a \in (-\infty; \frac{5}{16}\sqrt{7}; 9]$. We unite the results of both cases and find out that:

   - for $a \in (-\infty; -\frac{1+\sqrt{5}}{2}]$, $x_3$ is the solution of the equation;
   - for $a \in \left[ -\frac{1+\sqrt{5}}{2}; 1 \right]$, $x_1$ and $x_3$ are solutions to the equation;
   - for $a \in (1; +\infty)$, $x_2$ is the solution of the equation.

Therefore, the values of parameter $a$ are $(-\infty; -\frac{1+\sqrt{5}}{2}] \cup (1; +\infty)$. The presented solution definitely achieves the first three levels of learning - knowledge, understanding and application, the first three phases – understanding the problem, drawing up a plan and its implementation, and affecting the first two groups – student activities and educational content.

![Figure 2. Covered levels of training](image-url)

The question is, "Are the methodological possibilities of the task depleted or not?" Modern educational technologies provide the teacher with the opportunity to move to the upper levels in the Bloom Taxonomy. In the case at hand, with their help, the analytically obtained result may:
- Heuristically analyze, ie. to illustrate graphically;
- Presented through separate components;
- Make relevant links, conclusions, summaries.

Looking back, it reflects the analytical and logical efforts made, creates the fullest in the understanding of the task, enables "imitation" in other analogous situations. There is an opportunity to add educational value to results already achieved, especially when it comes to mature students at the end of their training. The interpretation of the roots of the equation as intersections of the graphs of two functions, each dependent on the parameter, the components of the task, the relations between them and the result itself are represented on the following graphs:

Figure 3. Function graph for \( a=3,5 \) \( f(x) = x|x + 2.5| - 2.5 \)

Figure 4. Function graph for \( a=1 \) \( f(x) = x|x + 2| \)

Figure 5. Function graph for \( a=-1 \) \( f(x) = x|x - 2| + 2 \)
The use of various electronic learning materials, in particular animated drawings, is almost a mass practice (bordering an epidemic). The question we want to put forward is that their use is methodologically sound and complete. Dangers in this respect are not so small - the transformation of the learning process into observation, the replacement of abstract thinking with results, the hyperbolisation of the role of information at the expense of creativity and others.

It is astonishing that Pólya made his insights as early as 1946 by saying: "The mathematics presented in Euclid style is a systematic, deductive science. But mathematics in the process of its creation is an experimental, inductive science. Both aspects are as old as old mathematics itself. But presenting the second aspect to students, teachers, and a broader audience is a new, up-to-date process "[2]. It is the experimental side of mathematics that becomes visible, accessible and methodically usable with the help of qualitatively new technologies to rise to the upper levels in the pyramid of knowledge. As an illustration of this process, we will look at the following task:

**Task 2:** Find the values of the parameter \(a\), for each of which the equation \(|x - 2| + b|2x + 1| = a\) has a solution for each value of \(b\).

**Solution:** Representation of the left side of the equation by the function graph \(f(x) = |x - 2| + b|2x + 1|\) and immediate observation of the behavior gives her the solution to the task. Quite experimentally set the fixed point \(A(-0.5; 2.5)\), then the solution of task \(a = 2.5\) is obvious.

![Figure 6. Function graph for b=2.5 \(f(x) = |x - 2| + 2.5|2x + 1|\)](image)

![Figure 7. Function graph for b=-0.5 \(f(x) = |x - 2| - 0.5|2x + 1|\)](image)
3 RESULTS

In the last decades the learning process has changed, depending on the demand and needs of learners. Assessing the cognitive dimension through cognitive dimension processes can be not only algorithmic but extend to: factual - assessing the level of knowledge of terminology and specific details and elements; procedural - depending on the knowledge of different algorithms, techniques and methods, and metacognitive dimension - for self-assessment as presented in Taxonomy for Learning, Teaching and Assessment Taxonomy: Bloom's Revision [5].

The above-mentioned tasks prove that appraisal of students can become not only algorithmic but also based on an analysis of whether learners can find the solution based on the drawings using different methods and techniques to evaluate their capabilities and ability to monitor and solve tasks. In education, the evaluation is done through a qualitative indicator that determines the degree of achievement of the expected learning outcomes [6]. This means that in a solution to a mathematical task, not only the end result of the solution, but the way to reach this solution and the use of the terms, the application of different techniques for solving, the use of analyzes.

The ways of presenting a mathematical assignment and the techniques to reach the decision to a large extent determine the perception and learning of the material. According to David Kolb, four learning styles are distinguished by a combination of two of the four phases in the learning cycle (Kolb, 2005):

- **The dreamer**: specific experience / observation and reflection
- **Thinker**: observation and meditation / abstract thinking
- **Decision Maker**: Abstract Thinking / Active Experimenting
- **Contractor**: Active experimentation / specific experience

![Figure 9. Kolb – learning styles](image)

\[ f(x) = |x - 2| - 2|2x + 1| \]
The correlation between cognitive processes and learning styles on the one hand and mathematics training on the other are studied extensively in Cognitive Processes in Mathematics Education, T. Tonova, 2013, [8]

In today’s dynamic and rapidly developing world, a teacher who wants to maintain a good level of teaching needs to adapt and use new methods, techniques and technologies to reach out to their students. He has to use different ways of presenting information and solving tasks, depending on the different learning styles.

Graphic presentation and e-learning materials can help to identify new practical applications and complement the integration degree group. On the other hand, the use of electronic systems serves for easier objective assessment and self-assessment.

Our practice, our observations and the shared experience of trainers and trainees lead us to the conclusion that the capabilities of modern educational technologies should be reflected in the goals of higher education training – analysis, synthesis, evaluation, assessment criteria and, respectively “look back”.

4 CONCLUSIONS

A global trend over recent decades has been the decline in the interest in mathematics and the natural sciences. This process is objective and profoundly conditioned, but one of the factors is the dissatisfaction of the students in the study of mathematics and the natural sciences. For modern pupils, the knowledge of mathematics remains locked in between the covers of the textbook and the true cognitive and applied character of the abstract and theoretical aspects of mathematics. We are convinced that the proper and skilful use of technology in education will reveal mathematics to students as an experimental, applied and challenging area of knowledge.

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